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The famous Blasius Boundary-Layer Flow Over the Flat Plate:

The Shooting Method for a Boundary-value Problem

The Blasius Boundary-layer (BL) flow over the flat plate is described by the following equations (see F.M. White, *Fluid Mechanics*, 3rd Edition, McGraw-Hill, 1994, p.397):

Diff.Eq.: $f'''+0.5ff''=0$ or $f_3 = -0.5 \cdot f_0 \cdot f_2$

where: $f_1 = \frac{u}{U} = \frac{d}{d\eta} f(\eta) = \frac{d}{d\eta} f_0$ $\quad \frac{v}{U} = Cv \cdot (\eta \cdot f_1 - f_0)$; $Cv = \sqrt{\frac{\nu}{xU}}$

$\eta = y \cdot \sqrt{\frac{U}{\nu \cdot x}}$ $\quad f_i$ ...is i-th derivative of f ($f_0=f$, $f_1=f'$, $f_2=f''$, $f_3=f'''$)

U=const free-stream velocity, also velocity at the edge of the BL

u(x,y)=velocity in stream-direction within the BL

v(x,y)=velocity in normal-to-stream(wall) direction within the BL

Boundary Conditions: v(0)=0 --> f(0)=0,
u(0)=0 --> f '(0)=0,
u(∞)=0 --> f '(∞)=1.

$$f := \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix} \begin{matrix} ..f_0(0) \\ ..f_1(0) \\ ..f_2(0) \end{matrix}$$

$f_2 = 0.5$ ...a guess

n-th order ODE may be transformed in a system of n=3 equations:

$f_1 = df_0/d\eta = f'$

$f_2 = df_1/d\eta = f''$

$f_3 = df_2/d\eta = f''' = -0.5 f_0 f_3$, where $f_0 = f$

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Solving a "Boundary-Value Problem" using the "Initial-Value Method": Guessing the unknown initial value of $f_2(0)$ until we obtain the known boundary value of $f_1(\infty)=1$

$$f := \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix} \quad \begin{matrix} ..f_0(0) \\ ..f_1(0) \\ ..f_2(0) \end{matrix} \qquad f_2 = 0.5 \quad \text{...a guess}$$

Diff.Eq. Derivatives:

$$D(\eta, f) := \begin{bmatrix} f_1 \\ f_2 \\ -0.5 \cdot f_0 \cdot f_2 \end{bmatrix} \quad ...f_3$$

MathCAD's buit-in function for solving diff. eqs. using the Runge-Kutta method (see HELP on next slide)

Solve using Runge-Kutta method: $Z := \text{rkfixed}(f, 0, 7, 700, D)$ $\quad \text{rows}(Z) = 701 \quad \text{cols}(Z) = 4$

Check the f(∞)=1??: $Z_{700,2} = 1.314$ ...NOT YET!

=1?

target := 1

We need to keep guessing, or use the "shooting method," see the corresponding program below:

$$\text{NOTE}: f'(\infty) \approx f'(7) = Z_{700,2}$$

$f_2 \leftarrow g1$

$t1 \leftarrow \text{rkfixed}(f, 0, 7, 700, D)_{700,2}$

$g2 \leftarrow 1.1 \cdot g1$

$f_2 \leftarrow g2$

$t2 \leftarrow \text{rkfixed}(f, 0, 7, 700, D)_{700,2}$

... actually the "*Shooting Method*" program is on the second slide from here

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SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

MathCAD's buit-in function for solving diff. eqs. using the Runge-Kutta method

rkfixed(y, x1, x2, npoints, D)

rkfixed returns a matrix in which: (1)the first column contains the points at which the solution is evaluated and (2)the remaining column contains the corresponding values of the solution and its first n-1 derivatives. **rkfixed** uses the fourth order Runge-Kutta method to solve first order differential equations.

…or of the function & its first (n-1) derivatives of n-th order ODE

Arguments:

- **y** must be a vector of n initial values.
- **x1**, **x2** are endpoints of the interval on which the solution to differential equations will be evaluated. Initial values in **y** are the values at **x1**.
- **npoints** is the number of points beyond the initial point at which the solution is to be approximated. This controls the number of rows (1 + **npoints**) in the matrix returned by **rkfixed**.
- D is an n-element vector-valued function containing first derivatives of unknown functions.

..or the first (n) derivatives of n-th order ODE

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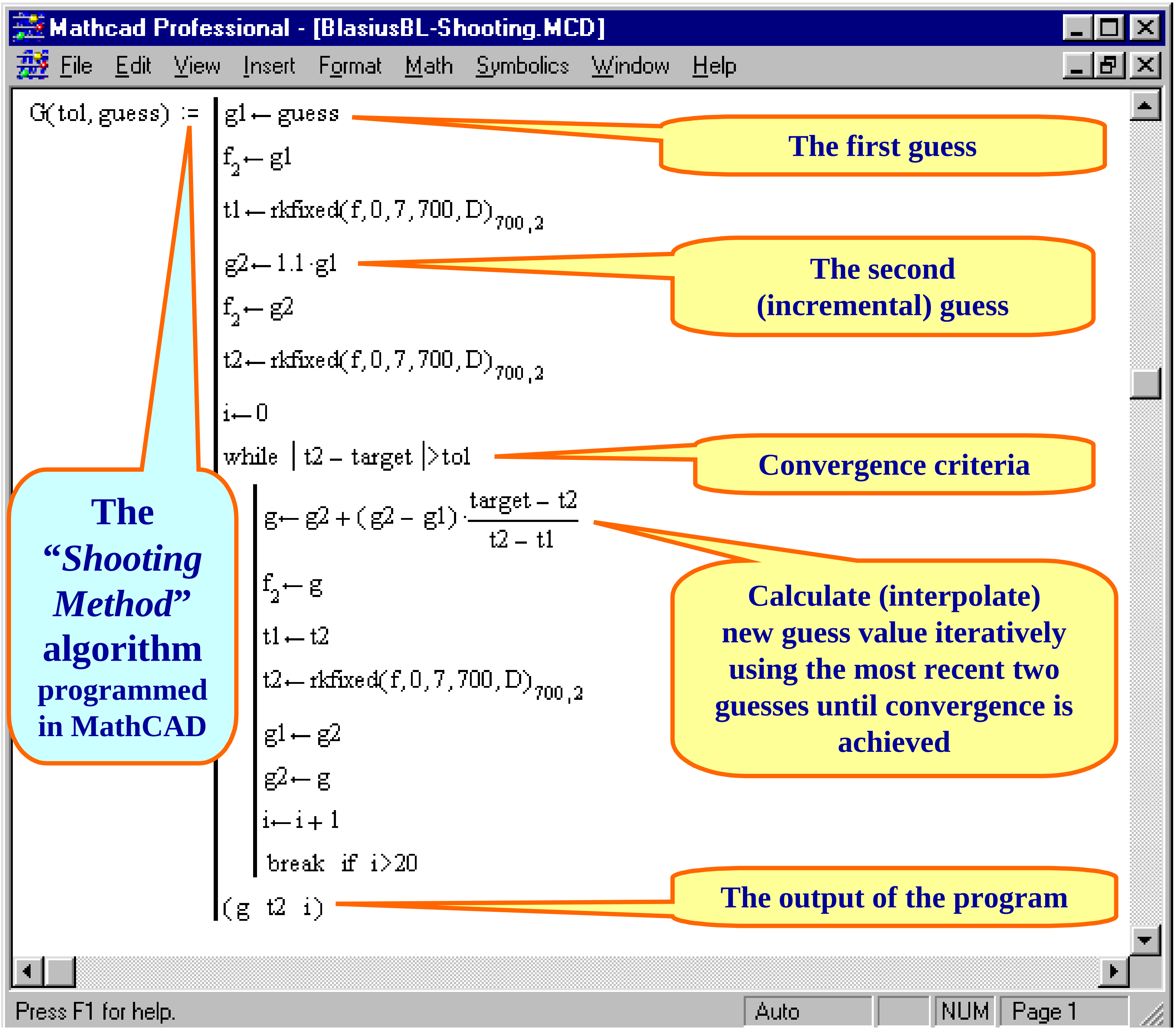

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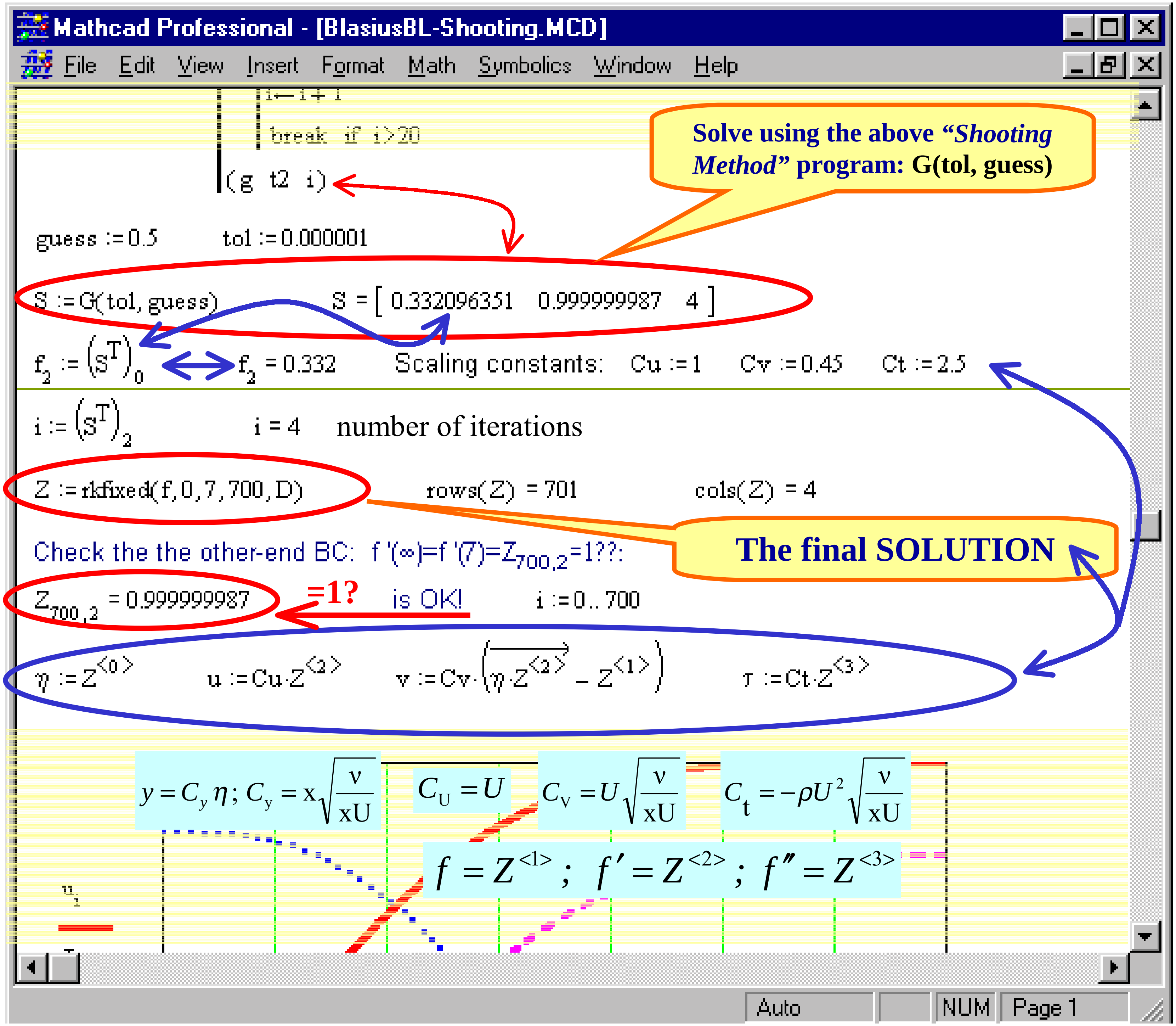

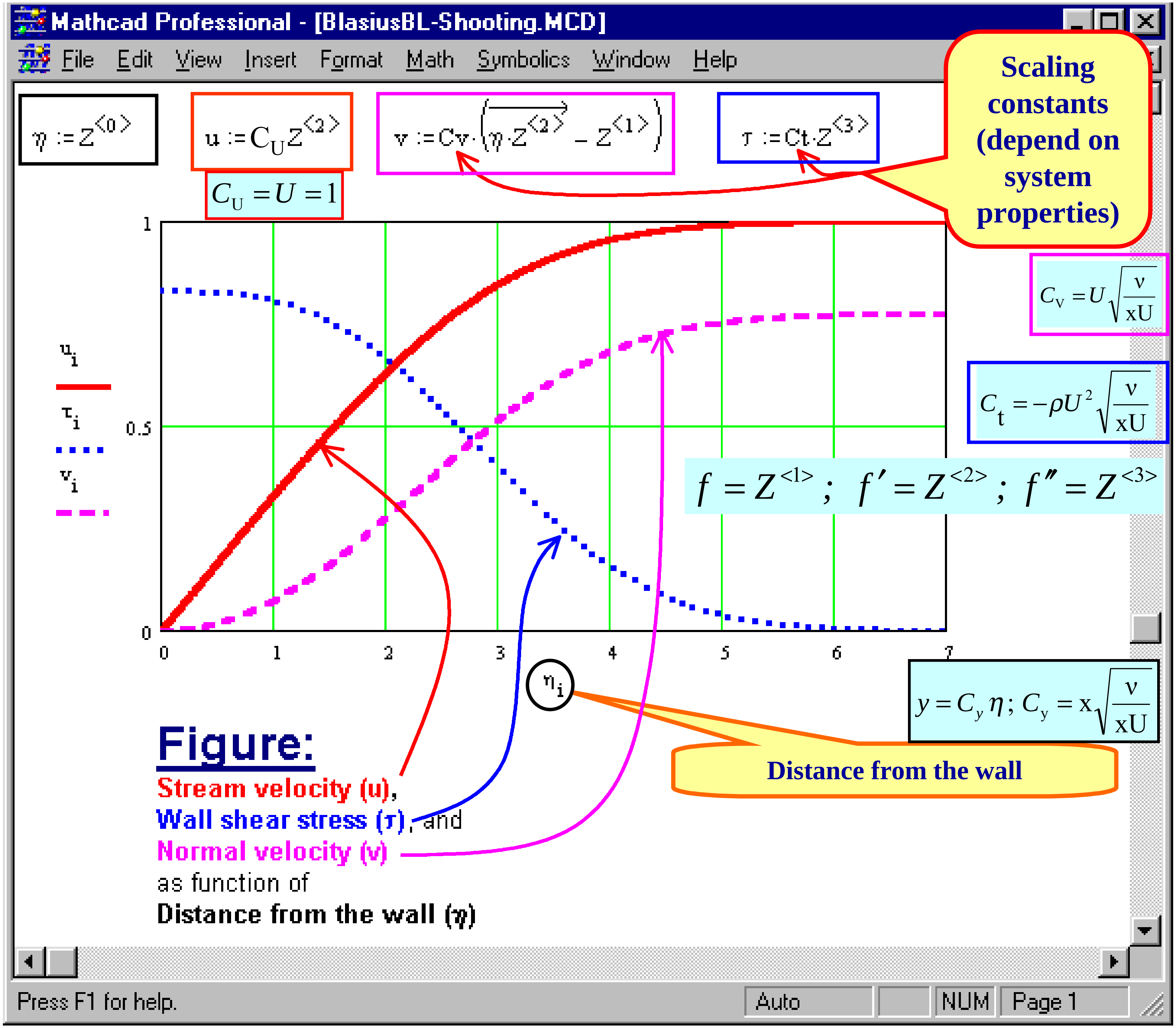

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Numerical values for the Blasius flow problem are tabulated below:

$j := 0..3 \quad k := 0..70 \quad ZZ_{k,j} := Z_{10 \cdot k,j}$

ZZ =

	0 (η)	1 (f)	2 (f ')	3 (f ")
0	0	0	0	0.3321
1	0.1	$1.6605 \cdot 10^{-3}$	0.0332	0.3321
2	0.2	$6.6418 \cdot 10^{-3}$	0.0664	0.332
3	0.3	0.0149	0.0996	0.3318
4	0.4	0.0266	0.1328	0.3315
5	0.5	0.0415	0.1659	0.3309
6	0.6	0.0597	0.199	0.3301
7	0.7	0.0813	0.2319	0.329
8	0.8	0.1061	0.2647	0.3274
9	0.9	0.1342	0.2974	0.3255
10	1	0.1656	0.3298	0.323
11	1.1	0.2002	0.362	0.3201
12	1.2	0.238	0.3938	0.3166
13	1.3	0.2789	0.4253	0.3126
14	1.4	0.323	0.4563	0.3079
15	1.5	0.3702	0.4868	0.3026

ZZ =

	0 (η)	1 (f)	2 (f ')	3 (f ")
49	4.9	3.1845	0.9899	0.0187
50	5	3.2836	0.9916	0.0159
51	5.1	3.3828	0.9931	0.0135
52	[illegible]	3.4822	0.9943	0.0113
53	5.3	[illegible]	0.9954	$9.5035 \cdot 10^{-3}$
54	5.4	[illegible]	[illegible]	[illegible]
55	5.5	[illegible]	[illegible]	[illegible]
56	5.6	3.8807	0.9976	$5.4303 \cdot 10^{-3}$
57	5.7	3.9804	0.9981	$4.4614 \cdot 10^{-3}$
58	5.8	4.0803	0.9985	$3.6472 \cdot 10^{-3}$
59	5.9	[illegible]	0.9988	$2.9667 \cdot 10^{-3}$
60	6	4.28	0.9991	$2.4011 \cdot 10^{-3}$
61	6.1	4.3799	0.9993	$1.9337 \cdot 10^{-3}$
62	6.2	4.4799	0.9994	$1.5495 \cdot 10^{-3}$

Within 1%

A good guess (solution) to get the 3rd Boundary Condition

Within 0.1%

BOUNDARY CONDITIONS

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