

# NEW EDUCATIONAL LAB: MEASUREMENT AND UNCERTAINTY EVALUATION OF NANOFLUID PARTICLE CONCENTRATION USING VOLUMETRIC FLASK METHOD

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## ABSTRACT

A new, simple educational lab with a contemporary theme, to measure nanofluid particle concentration, is developed. Nanofluids are colloidal suspensions of nanosize particles, including fibers or composites, in base fluids. They may be produced by direct mixing of measured quantities of nanoparticles and base fluids, thus concentration is predetermined. However, if nanoparticles are deposited into a fluid in a process without being able to quantify exact amount of nanoparticles, or if nanofluid of unknown particle concentration is to be quantified, then there is a need to develop a technique of measuring nanoparticle concentration in a nanofluid. A new, simple educational lab, with related error uncertainty analysis, is developed and tested using a volumetric flask method. The method is based on measured or known density of nanoparticles (i.e. if their composition is known) and measurement of densities of base fluid and nanofluid using precise analytic balance and volumetric flask with specified volume and its accuracy. Based on the conservation of mass and assumed conservation of volume of the colloidal mixture components, the equations for mass- and volume-concentration of nanoparticles in nanofluid are developed as well as for the related measurement results uncertainties. The instrumentation and method is first calibrated by measuring known density of distilled water and known nanofluid concentrations, the latter made by direct mixing of predetermined quantities of CuO or Al<sub>2</sub>O<sub>3</sub> nanoparticles in distilled water up to 25% mass concentration. Deviations of measured from known values were within determined uncertainties, i.e. less than 0.6% for density and about 1% for particle concentration in nanofluids with 50 mL volumetric flask (note that 1% is 25% relative to the 4% measured concentration, for example). Therefore, for small nanoparticle concentrations in nanofluids (several percents) the concentration measurement uncertainty is relatively large; however, for higher concentrations it is acceptable. This method is being further improved and may be used if nanofluids are concentrated by evaporation of the base fluid or other means, or for comparative measurements where absolute accuracy is less important. This simple measurement method illustrates importance of measurement uncertainty evaluation and it is very easy to introduce in any related course with laboratory component.

*Keywords:* nanoparticles, base fluid, nanofluid, density, uncertainty, volumetric flask

## NOMENCLATURE

$C_m$	Mass concentration of nanoparticles in nanofluid mixture [1, unitless]
$C_{mB}$	Mass concentration of nanoparticles with respect to base fluid [1]
$C_{mB,st}$	Known, standard mass concentration of nanoparticles in base fluid [1]
$C_V$	Volumetric concentration of nanoparticles in nanofluid mixture [1]
$C_{VB}$	Volumetric concentration of nanoparticles with respect to base fluid [1]
$d_{r\%}$	Percentage deviation in the measured density [%]
$m_0$	Mass of empty volumetric flask [g or kg]
$m_T$	Mass of volumetric flask with fluid [g or kg]
$m_T - m_0$	Mass of the fluid [g or kg]
$T$	Fluid temperature [ $^{\circ}C$ ]
$u_d$	Design state uncertainty [in relevant measurement units (RMU*)]
$u_c$	Calibration uncertainty [RMU*]
$u_{Cm}$	Uncertainty of mass concentration of nanoparticles [ $kg/m^3$ ]
$u_{CB}$	Uncertainty in concentration due to uncertainty of base fluid density [1]
$u_{Cn}$	Uncertainty in concentration due to uncertainty of nanofluid density [1]
$u_{CP}$	Uncertainty in concentration due to uncertainty of nanoparticles' density [1]
$u_m$	Uncertainty in measuring the mass [mg or kg]
$u_{mT}$	Uncertainty in total flask mass (empty flask mass + fluid mass) [mg or kg]
$u_{m0}$	Uncertainty in empty flask [mg or kg]
$u_0$	Zero state uncertainty (half of an instrument resolution) [RMU*]
$u_T$	Uncertainty in temperature [ $^{\circ}C$ ]
$u_V$	Uncertainty in measuring the volume [mL]
$u_r$	Uncertainty in measuring the density of fluid [ $kg/m^3$ ]
$u_{rn}$	Uncertainty in density of nano fluid [ $kg/m^3$ ]
$u_{rB}$	Uncertainty in density of base fluid [ $kg/m^3$ ]
$u_{rP}$	Uncertainty in density of nanoparticles [ $kg/m^3$ ]
$u_{rT}$	Uncertainty in density due to temperature uncertainty [ $kg/m^3$ ]
$V$	Standard volume of the flask [mL]
$V_p$	Volume of nanoparticles [mL]
$\rho_n$	Density of nanofluid [ $kg/m^3$ ]
$\rho_m$	Measured density of water [ $kg/m^3$ ]
$\rho_{ref}$	Reference density of water [ $kg/m^3$ ]

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\* in relevant measurement units (RMU)

$\rho_{T_1}$	Base fluid density at temperature $T_1$ [ $kg/m^3$ ]
$\rho_{T_2}$	Base fluid density at temperature $T_2$ [ $kg/m^3$ ]

### *Selected Subscripts*

$B$	Base fluid (water)
$n$	Nanofluid
$P$	Nanoparticles

## 1. INTRODUCTION

A new, simple educational lab with a contemporary theme to measure nanofluid particle concentration using volumetric flask, is developed. This method, using a standard volumetric flask and analytic balance, is based on the conservation of mass and assumed conservation of component volumes in the mixture. Further research and testing are needed to account for possible small nanofluid mixture volume change, and it is beyond the scope of this paper. The volumetric flask is calibrated using distilled water in order to determine the uncertainty in measurements. Nanofluids are an innovative class of fluids with enhanced properties made of nanosize particles or fibers suspended in common, base fluid. One of the methods of producing nanofluid is by direct mixing of measured quantities of nanoparticles and base fluid (fluid used for preparing nanofluids), thus concentration is predetermined. However, if nanoparticles are deposited into a fluid in a process without being able to quantify exact amount of nanoparticles (which is often the case), or if nanofluid of unknown particle concentration is to be quantified, then there is a need to develop a technique for measuring the nanoparticle concentration in a nanofluid. The same method could be used for measurements of particle concentration in any particle-fluid mixture. The method also illustrates the simple use of basic instrumentation and fundamental conservation laws in physical sciences, as well as relevant error uncertainty analysis needed in engineering and scientific measurements.

The volumetric flask method was tested with nanofluids made in our laboratory. Nanofluids with predetermined mass concentrations up to 25% were made using direct mixing method with commercial nanoparticles of aluminum-oxide (alumina,  $Al_2O_3$ ) and copper-oxide ( $CuO$ ), and distilled water as the base fluid. The density of base fluid and nanofluid were then independently determined using the volumetric flask with known standard volume and measuring the mass of each fluid with analytic balance. The densities of nanoparticles were given by manufacturer and verified in references based on their composition. Then, using the densities of nanoparticles, base fluid and nanofluid, the nanoparticle mass-concentration and volumetric-concentration in the nanofluid were determined along with relevant error uncertainties, using developed procedure in this paper. Uncertainty analysis of experimentally measured values was verified by comparing the uncertainties obtained with deviations of measured data from known values. The developed lab is simple, but contemporary application and required rigor in experimental accuracy and detailed uncertainty analysis, should enhance motivation and educational values.

## 2. THEORY

The relations for mass and volume concentration of nanoparticles in base fluid and/or nanofluid have been developed based on conservation of mass and assumed conservation of mixture volume of its components [1]. The volume of a mixture is not exactly the sum of the volume of its components due to intermolecular and inter-particle forces and related displacements; however the volume change is assumed negligible due to the nanoparticle size (10-100 nm) being order(s) of magnitude larger than the liquid molecular size, which is also confirmed by experiments. Further research and testing are needed to account for rather small nanofluid mixture volume change, and it is beyond the scope of this paper. Two volumetric flasks (25±0.03 mL and 50±0.05 mL) of Class-A type have been used for measurement of fluid volume. The volumetric flask (including the measurement method) was initially calibrated using distilled water and the results were compared to reference densities [2]. A digital analytical balance (±0.1mg resolution) is used to measure the mass of the volumetric flask with and without fluid in it. A digital thermometer (±0.1 °C resolution) was used for measuring the temperatures of the fluid and the surroundings. The volumetric flask method was initially calibrated using distilled water and the results were compared to reference densities [2].

### *Fluid Density Measurement:*

Volumetric flask is used to measure fluid or fluid-mixture mass (total flask with fluid minus empty flask mass,  $m_T - m_o$ ) in calibrated flask volume ( $V$ ) and thus determine base-fluid ( $\mathbf{r}_B$ ), or nanofluid mixture density ( $\mathbf{r}_n$ ) [3]. Then, in general, density,  $\mathbf{r}$ , and its uncertainty,  $u_r$ , may be determined as [4,5]:

$$\mathbf{r} = \frac{m_T - m_o}{V} = \mathbf{r}(m_T, m_o, V) \quad (= \mathbf{r}_n \text{ or } = \mathbf{r}_B) \quad (1)$$

$$u_r = \sqrt{\left(\frac{\partial \mathbf{r}}{\partial m_T} u_{mT}\right)^2 + \left(\frac{\partial \mathbf{r}}{\partial m_o} u_{m_o}\right)^2 + \left(\frac{\partial \mathbf{r}}{\partial V} u_V\right)^2} = \sqrt{\left(\frac{1}{V} u_{mT}\right)^2 + \left(\frac{1}{V} u_{m_o}\right)^2 + \left(\frac{m_T - m_o}{V^2} u_V\right)^2} \quad (2)$$

Most of the uncertainty comes from uncertainty in flask volume since the mass accuracy measured with analytic balance is much higher. Furthermore, since the uncertainty due to volume, see Eq. (2) depends on net-mass (which is proportional to volume) and inversely proportional to volume squared, then the density uncertainty will increase with the flask volume uncertainty, but decrease linearly with the flask volume increase. Note that the above Eq. (2) represents the uncertainty in measured density at a given temperature, and does not account for temperature uncertainty (the latter will be included in Eq. 12).

### *Nanofluid Concentration Measurement:*

If we measure density of a nanofluid ( $\mathbf{r}_n$ ) of unknown concentration, but known type of base fluid (known or measured  $\mathbf{r}_B$ ) and known nanoparticle material ( $\mathbf{r}_p$ ), using the above method, then the conservation of mass and volume of the base-fluid and nanoparticle mixture will be [1]:

$$V = V_n = V_B + V_P \quad (3)$$

$$m_T - m_o = m_n = m_B + m_P = \mathbf{r}_B V_B + \mathbf{r}_P V_P \quad (4)$$

From Eqs. (3 & 4) we could express volume fractions of the base fluid and nanoparticles in the flask volume, i.e.:

$$\frac{V_B}{V} = \frac{\mathbf{r}_P - \mathbf{r}_n}{\mathbf{r}_P - \mathbf{r}_B} \quad (5)$$

$$C_V = \frac{V_P}{V} = \frac{\mathbf{r}_n - \mathbf{r}_B}{\mathbf{r}_P - \mathbf{r}_B} \quad (6)$$

The above, Eq. (6), is the volumetric concentration ( $C_V$ ) of nanoparticles in nanofluid, while the mass concentration ( $C_m$ ) of nanoparticles in nanofluid will be:

$$C_m = \frac{m_P}{m_n} = \frac{\mathbf{r}_P}{\mathbf{r}_n} C_V = \frac{\mathbf{r}_P}{\mathbf{r}_n} \cdot \frac{\mathbf{r}_n - \mathbf{r}_B}{\mathbf{r}_P - \mathbf{r}_B} = C_m(\mathbf{r}_B, \mathbf{r}_P, \mathbf{r}_n) \quad (7)$$

The corresponding volumetric ( $C_{VB}$ ) and mass ( $C_{mB}$ ) concentrations of nanoparticles with respect to the base fluid volume will be, respectively:

$$C_{VB} = \frac{V_P}{V_B} = C_V \frac{V}{V_B} = \frac{\mathbf{r}_n - \mathbf{r}_B}{\mathbf{r}_P - \mathbf{r}_n} \quad (8)$$

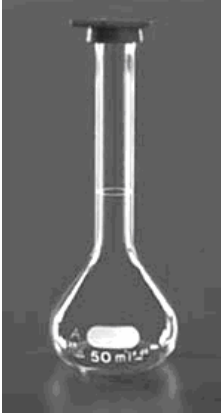
$$C_{mB} = \frac{m_P}{m_B} = \frac{\mathbf{r}_P}{\mathbf{r}_B} C_{VB} = \frac{\mathbf{r}_P}{\mathbf{r}_B} \cdot \frac{\mathbf{r}_n - \mathbf{r}_B}{\mathbf{r}_P - \mathbf{r}_n} \quad (9)$$

The uncertainty ( $u_{Cm}$ ) of mass concentration ( $C_m$ ) of nanoparticles in nanofluid is calculated using Eq. (7), and could be calculated in similar manner for the other types of concentrations, namely:

$$\begin{aligned} u_{Cm} &= \sqrt{\left(\frac{\partial C_m}{\partial \mathbf{r}_B} u_{rB}\right)^2 + \left(\frac{\partial C_m}{\partial \mathbf{r}_P} u_{rP}\right)^2 + \left(\frac{\partial C_m}{\partial \mathbf{r}_n} u_{rn}\right)^2} = \\ &= \sqrt{\underbrace{\left(\frac{\mathbf{r}_P}{\mathbf{r}_n} \cdot \frac{\mathbf{r}_P - \mathbf{r}_n}{(\mathbf{r}_P - \mathbf{r}_B)^2} u_{rB}\right)^2}_{u_{CB}} + \underbrace{\left(\frac{\mathbf{r}_B}{\mathbf{r}_n} \cdot \frac{\mathbf{r}_n - \mathbf{r}_B}{(\mathbf{r}_P - \mathbf{r}_B)^2} u_{rP}\right)^2}_{u_{CP}} + \underbrace{\left(\frac{\mathbf{r}_B \mathbf{r}_P}{\mathbf{r}_n^2 (\mathbf{r}_P - \mathbf{r}_B)} u_{rn}\right)^2}_{U_{Cn}}} \end{aligned} \quad (10)$$

### 3. VOLUMETRIC FLASK CALIBRATION

*Fluid Density Measurement:*



**Figure 1:**  
Volumetric flask.

Fluid mass,  $m_T - m_o$ , in the standard volume ( $V$ ) of the volumetric flask, is measured using the analytic balance (where  $m_T$  is the total mass of the fluid and flask,  $m_o$  is mass of empty flask). Then, measured fluid density ( $\mathbf{r}_m$ ) is determined using Eq. (1). The percentage deviation in the measured fluid density from the reference fluid density is determined using Eq. (11). The measured densities of distilled water ( $\mathbf{r}_m$ ) are presented in Table 1 and Figure 2 along with the corresponding reference values ( $\mathbf{r}_{ref}$ ) [2]. Measured density, see Eq. 1 above, is:

$$\mathbf{r}_m = \frac{m_T - m_o}{V}$$

The percentage (%) deviation in density is:

$$d_{r\%} = \frac{\mathbf{r}_m - \mathbf{r}_{ref}}{\mathbf{r}_{ref}} \cdot 100\% \quad (11)$$

*Uncertainty in Density:*

As part of the volumetric flask calibration and to justify the water density deviations from reference values, the relevant uncertainty analysis has been performed. The uncertainty in density ( $u_r$ ) is calculated using mass, volume, and temperature uncertainties of the fluid density (Eq. 12), obtained by expanding Eq. (2), to account for the density temperature-dependence and uncertainty in temperature measurements (see last term under root in Eq. 12), since the measured density values are compared with reference values at the measured temperature. The results are presented in Table 1.

$$u_r = \sqrt{2\left(\frac{u_m}{V}\right)^2 + \left(\frac{m_T - m_o}{V^2} u_v\right)^2 + \left(\frac{\partial \mathbf{r}}{\partial T} u_T\right)^2} \quad (12)$$

**Table 1: Calibration of volumetric flask: distilled water density measurements.**

No.	Input data				Results				
	Flask (V) [mL]	Temp (T) [°C]	$m_o$ [g]	$m_T$ [g]	$\mathbf{r}_m$ [kg/m <sup>3</sup> ] (1)	$\mathbf{r}_{ref}$ [kg/m <sup>3</sup> ] [2]	$d_{r\%}$ (11)	$u_r\%$ (12)	Density range ( $\mathbf{r}_m - u_r$ ) -- ( $\mathbf{r}_m + u_r$ ) [kg/m <sup>3</sup> ]
1	25	20.4	19.9900	44.8802	995.652	998.123	0.05 %	0.8%	987.661 -- 1003.682
2	50	19.9	35.6028	85.4202	996.358	998.227	0.18 %	0.6%	990.358 -- 1002.358
3	50	19.8	35.6121	85.4468	996.708	998.247	0.154%	0.6%	990.727 -- 1002.688

**Note:**  $u_m = 0.21$  mg;  $u_v = 0.2$  mL for 25 mL flask and 0.3 mL for 50 mL flask;  $u_T = 0.2$  °C;  $u_{rT} = 0.015$  kg/m<sup>3</sup> (14).

The same Equation (12) can be used for uncertainty in density of nanofluid ( $u_{rn}$ ) and base fluid ( $u_{rB}$ ).

*Uncertainty in Mass:*

The uncertainty in density, Eq. (12), depends on uncertainty in mass,  $u_m$ , which in turn depends on resolution and accuracy of the analytic balance used. The resolution uncertainty,  $u_0 = 0.05 \text{ mg}$  (i.e. half the resolution of  $0.1 \text{ mg}$ ), and its estimated calibration uncertainty  $u_c = 0.2 \text{ mg}$  (estimated to be twice the resolution). Thus, the design state uncertainty,  $u_d = 0.21 \text{ mg} = 0.21 \cdot 10^{-6} \text{ kg}$ , has been calculated using Eq. (13).

$$u_d = \sqrt{u_0^2 + u_c^2} \quad (13)$$

*Uncertainty in Volume:*

In addition to the volumetric flask, Class-A type calibration accuracy of  $\pm 0.03 \text{ mL}$  and  $\pm 0.05 \text{ mL}$  for  $25 \text{ mL}$  and  $50 \text{ mL}$  flasks, respectively, the meniscus errors due to fluid-to-flask wall adhesion causes uncertainty in measurements also. Therefore, much larger uncertainty in volume  $u_v$  in Eq. (12) has been estimated as  $0.2 \text{ mL}$  for  $25 \text{ mL}$  flask and  $0.3 \text{ mL}$  for  $50 \text{ mL}$  flask (also to account for assumed mixture volume conservation).

*Uncertainty in Temperature:*

The temperature is measured using a digital temperature probe with  $0.1 \text{ }^\circ\text{C}$  resolution. The zero stage  $u_0$  and calibration uncertainty  $u_c$  are assumed to be half of and double the resolution of the instrument, i.e. equal to  $0.05 \text{ }^\circ\text{C}$  and  $0.2 \text{ }^\circ\text{C}$ , respectively. The temperature uncertainty,  $u_T$ , in Eq. (12) corresponds to the design stage uncertainty  $u_d = 0.2 \text{ }^\circ\text{C}$ , calculated using Eq. (13).

*Uncertainty in Density Due to Temperature Uncertainty:*

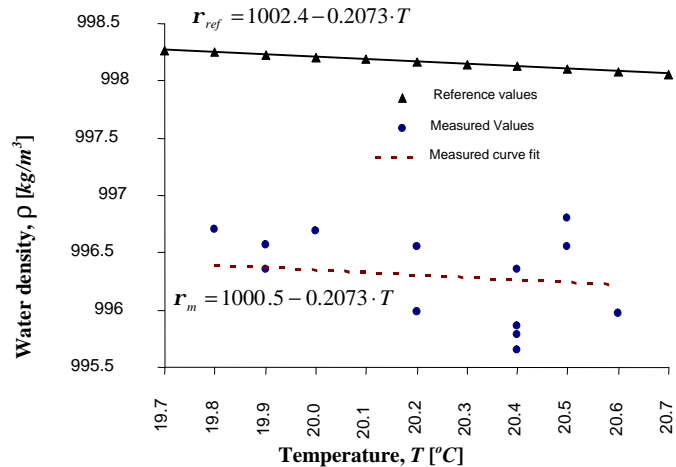
The temperature also contributes to the uncertainty in the density since it is measured at some temperature and compared with the corresponding reference value at that temperature. So the uncertainty in density because of temperature,  $u_{rT}$  is found to be  $0.015 \text{ kg/m}^3$  using Eq. (14), where density dependence on temperature is approximated with the corresponding finite difference, using the reference table values [2] closest to the measured temperature (e.g.,  $19.5 \text{ }^\circ\text{C}$  and  $20.5 \text{ }^\circ\text{C}$  for measured temperature of  $20 \text{ }^\circ\text{C}$ ).

$$u_{rT} = \frac{\partial \mathbf{r}}{\partial T} \cdot u_T \approx \frac{\mathbf{r}_{T1} - \mathbf{r}_{T2}}{T_1 - T_2} \cdot u_T \quad (14)$$

The uncertainty in mass ( $u_m$ ), volume ( $u_v$ ) and temperature ( $u_T$ ) are substituted in Eq. (12) to determine the uncertainty in density ( $u_r$  in Table 1).

*Overall Bias Error:*

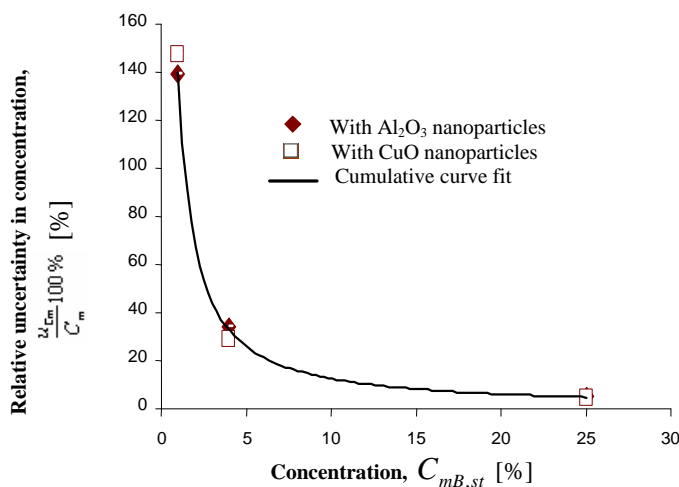
The measured calibration values were curve-fitted with a line using the slope corresponding to the well-established reference data [2], see Figure 2. An overall bias error of  $0.8 \text{ kg/m}^3$  (or about 0.08 %) was found during the calibration of density measurements. Bias error could be corrected in data analysis, but it has not been done here due to rather small value of the bias error and limited calibration data. Note very large magnification of the density scale on Figure 2.



**Figure 2: Water density at different temperatures: measured data are curve-fitted using the slope corresponding to the reference values ( $T[^\circ\text{C}]$ ) [2].**

**4. NANOFLUIDS CONCENTRATION MEASUREMENT**

Nanofluids have been prepared by direct mixing method using pre-measured nanoparticles’ mass and the base fluid (water) mass. Nanoparticles of  $\text{CuO}$  or  $\text{Al}_2\text{O}_3$  have been used for preparing 1%, 4% and 25 % concentrations by mass. The density of nanofluids prepared ( $r_n$ ) were measured using the above method (see Eq. 1) as if they were unknown. Thus, the mass and volume concentrations of nanoparticles in nanofluids were calculated using Eqs. (6-9), and the corresponding uncertainties using Eq. (10). All data are presented in Table 2 with quantities defined in the text and



**Figure 3: Uncertainty of mass concentration of nanoparticles in base fluid as function of concentration (for 50 mL volumetric flask).**

and Nomenclature. Variation of relative uncertainty with the particle concentration is depicted in Figure 3 using data from Table 2. Note that relative uncertainty in mass concentration is decreasing inversely with the concentration increase, since the mass concentration uncertainty for the given volumetric flask and other similar conditions is approximately constant (about 1%, see Table 2). Thus, the relative uncertainty is prohibitively high for mass concentrations smaller than the concentration uncertainty.

**Table 2: Nanofluid particle concentration and uncertainty data**

No.	Input data					Results										
	Flask (V) [mL]	nanofluid	$C_{mB,st}$ %	$m_0$ [g]	$m_T$ [g]	$r_n$ [kg/m <sup>3</sup> ]	$V_p$ [mL]	$C_v$ %	$C_m$ %	$C_{mB}$ %	$u_{rB} = u_{rn}$ [kg/m <sup>3</sup> ]	$\frac{u_{CB}}{u_{Cm}} 100\%$ %	$\frac{u_{CP}}{u_{Cm}} 100\%$ %	$\frac{u_{Cb}}{u_{Cm}} 100\%$ %	$u_{Cm}$ %	$\frac{u_{Cm}}{C_m} 100\%$
	Equation No.					(1)	(6)	(6)	(7)	(9)	(12)	(10)*			(10)	
1	25	Al <sub>2</sub> O <sub>3</sub>	1	22.7772	48.2571	1019.18	0.017	0.068	0.266	0.267	8.15	70.75	0.12	70.66	1.51	570.1
2	25	CuO	1	22.7838	48.7844	1021.56	0.10	0.417	2.60	2.66	8.32	71.34	0.73	70.06	1.32	50.8
3	50	Al <sub>2</sub> O <sub>3</sub>	1	36.2551	86.3948	1002.78	0.102	0.20	0.81	0.815	6.02	70.85	0.48	70.56	1.12	139.3
4	50	CuO	1	35.7129	85.8360	1002.46	0.052	0.104	0.678	0.682	6.02	70.87	0.24	70.54	0.99	147.3
5	50	Al <sub>2</sub> O <sub>3</sub>	1	35.6120	85.6791	1001.35	0.077	0.155	0.617	0.620	6.01	70.81	0.36	70.60	1.13	182.8
6	50	CuO	1	35.7354	85.8631	1002.55	0.053	0.106	0.689	0.693	6.02	70.87	0.25	70.54	0.99	144.9
7	50	Al <sub>2</sub> O <sub>3</sub>	4	35.5853	86.6745	1021.77	0.421	0.843	3.276	3.359	6.13	71.27	1.97	70.11	1.11	34.0
8	50	CuO	4	35.6903	86.9891	1025.97	0.266	0.532	3.370	3.469	6.16	71.53	1.24	69.86	0.99	29.2
9	50	Al <sub>2</sub> O <sub>3</sub>	25	35.6891	94.3083	1172.39	2.95	5.908	20.00	23.53	7.03	73.560	13.09	66.46	1.02	5.1
10	50	CuO	25	36.2448	95.9304	1193.69	1.793	3.586	19.49	23.35	7.16	75.36	7.80	65.26	0.91	4.7

\*) Partial uncertainties are combined using the Root-Sum-Square (RSS) rule.

**Note 1:** Density of Al<sub>2</sub>O<sub>3</sub> nanoparticles is 3970 kg/m<sup>3</sup> and of CuO nanoparticles is 6490 kg/m<sup>3</sup>,  $u_{rP} = 79.4$  kg/m<sup>3</sup> for Al<sub>2</sub>O<sub>3</sub> and 129.8 kg/m<sup>3</sup> for CuO (2% of density values).

**Note 2:** The relative uncertainty in mass concentration is decreasing inversely with the concentration increase (see the last column above), since the mass concentration uncertainty,  $u_{Cm}$ , (for the given volumetric flask and other similar conditions) is approximately constant (about 1%). Thus, the relative uncertainty is prohibitively high for mass concentrations smaller than the concentration uncertainty. Uncertainty analysis shows (see unacceptable shaded values in last column) that this method is not accurate at all for small concentrations of nanoparticles in nanofluids (1% in this case) as well as for small volume of volumetric flask (25 mL in this case). However, for 50 mL volumetric flask and larger nanoparticle concentrations (25%) the relative uncertainties are rather small (about 5%). The results presented here illustrate importance of evaluation of measured results by determining the corresponding measurement uncertainties. This method is being further improved and may be used if nanofluids are concentrated by evaporation of the base fluid or other means, or for comparative measurements where absolute accuracy is less important.

## 5. CONCLUSION

It has been observed by the measurements, see Table 1 & 2, that the 50 mL volumetric flask is more accurate than the 25 mL volumetric flask for measuring the density and mass concentration of nanoparticles in all tested nanofluids. With the same flask, the uncertainty in measured mass concentration is higher for nanofluids with lower nanoparticle concentrations (i.e. 1 % by weight than for 4 and 25 % by weight). For higher concentrations (4 % and 25 % by weight), the uncertainties are within acceptable range, i.e. about 0.6 % for density (Table 1) and about 1% for nanoparticle mass concentration in nanofluids with 50 mL volumetric flask (1% is still high, i.e. it is 25 % relative to the 4% measured concentration, for example, see Table 2). Most of the uncertainty comes from the uncertainty in volume of the volumetric flask (compare values in the tree columns before the last two columns in Table 2). Furthermore, the relative concentration uncertainty is inversely decreasing with the concentration increase. For small concentrations (several percents) the measurement uncertainty is relatively large and unacceptable, however, for higher concentrations (above 10 %) it is acceptable. Therefore, for lower concentrations, larger volumetric flasks are recommended. This method is being further improved and may be used if nanofluids are concentrated by evaporation of the base fluid or other means, or for comparative measurements where absolute accuracy is less important.

Because of its simplicity, this experimental method can be easily adapted in any related course with laboratory component. The educational values are enhanced by detailed error uncertainty analysis. It demonstrates shortcomings of the method for certain values of the measured quantities and importance of full understanding of physical concept, instrumentations and measurement methods.

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