Ch. 2

Static and Dynamic Characteristics of Signals

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  Representing a step/square wave with Fourier Series

Classification of Waveforms

TABLE 2.1 Classification of Waveforms

I. Static

1. $y(t) = A_0$

II. Dynamic

Periodic waveforms

Simple periodic waveform

2. $y(t) = A_0 + C\sin(\omega t + \phi)$

Complex periodic waveform

3. $y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$

Aperiodic waveforms

Step[a]

4. $y(t) = A_0 U(t) = A_0 \quad \text{for } t > 0$

Ramp

5. $y(t) = Kt \quad \text{for } 0 < t < t_f$

Pulse[b]

6. $y(t) = A_0 U(t) - A_0 U(t - t_1)$

III. Nondeterministic waveform

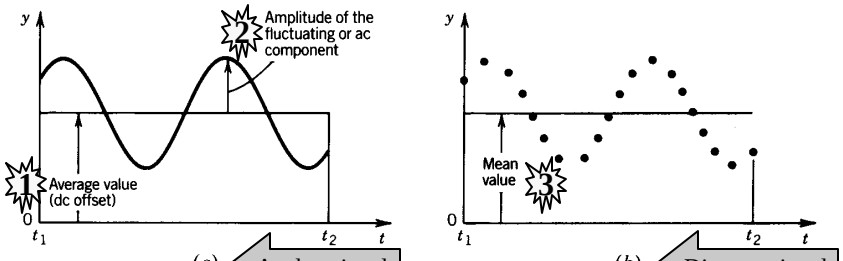
7. $y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$

[a] $U(t)$ represents the unit step function, which is zero for $t < 0$ and 1 for $t \geq 0$.

[b] t_1 represents the pulse width.

Dynamic signal

FIGURE 2.6 Analog and discrete representations of a dynamic signal.

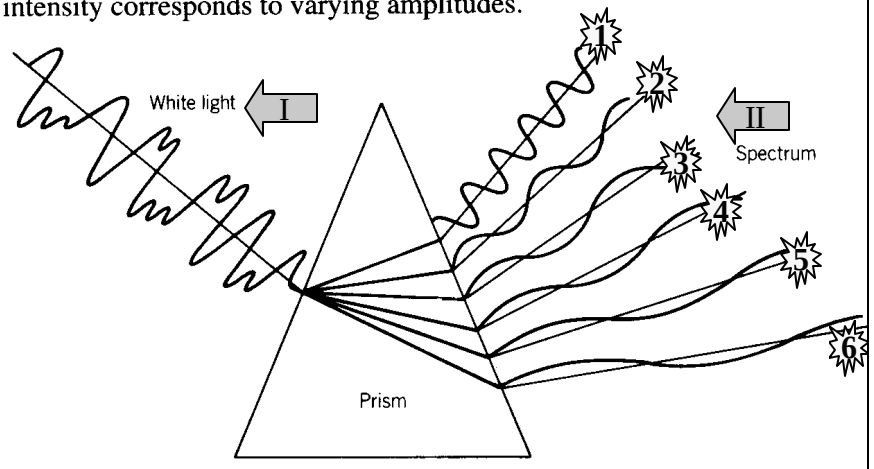


Average value: $\bar{y} \equiv \dfrac{\int_{t_1}^{t_2} y(t)\,dt}{\int_{t_1}^{t_2} dt}$

5 Root-Mean-Square average: $\int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} [I(t)]^2 R\,dt \qquad y_{\text{rms}} \equiv \sqrt{\dfrac{1}{t_2 - t_1} \int_{t_1}^{t_2} y^2\,dt}$

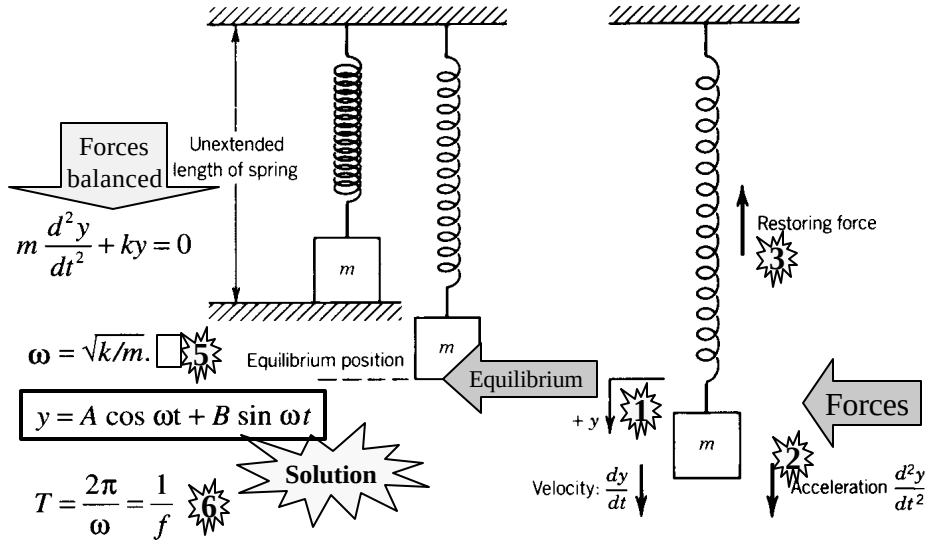
Separation of light into spectrum

FIGURE 2.9 Separation of white light into its color spectrum. Color corresponds to a particular frequency or wavelength, while light intensity corresponds to varying amplitudes.



Spring-mass system

FIGURE 2.10 Spring-mass system.



Fourier Coefficients

The coefficients of a trigonometric series representing a function of frequency ω are given by the Euler formulas,

$$1 \quad A_0 = \frac{1}{T}\int_{-T/2}^{T/2} y(t)\,dt$$

$$2 \quad A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos n\omega t\,dt \qquad (2.17)$$

$$3 \quad B_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\sin n\omega t\,dt$$

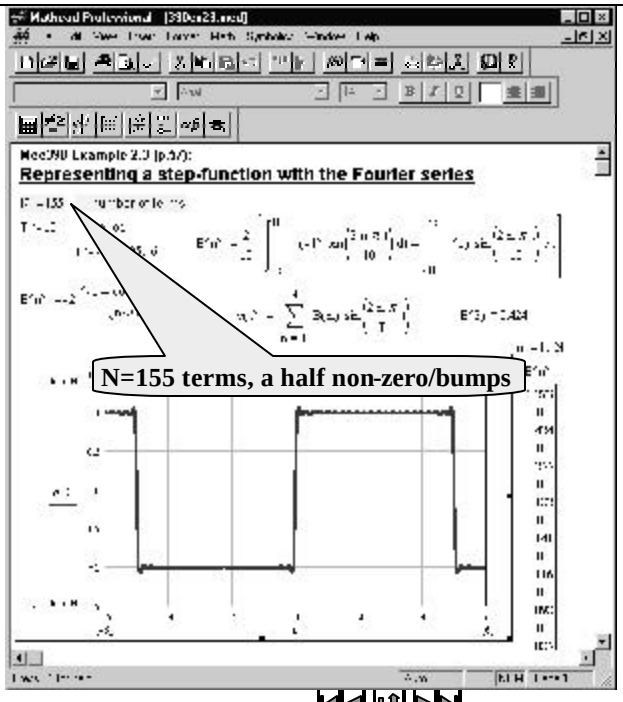
where $n = 1, 2, 3, \ldots$, and $T = 2\pi/\omega$ is the period of $y(t)$. The trigonometric series that results from these coefficients is a Fourier series, and may be written as

Any periodic function

$$y(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right) \qquad (2.18)$$

Its Fourier series

MathCAD



MathCAD Example:

Representing a step/square wave with Fourier Series

The End

Thank you for your attention