Strain Measurement

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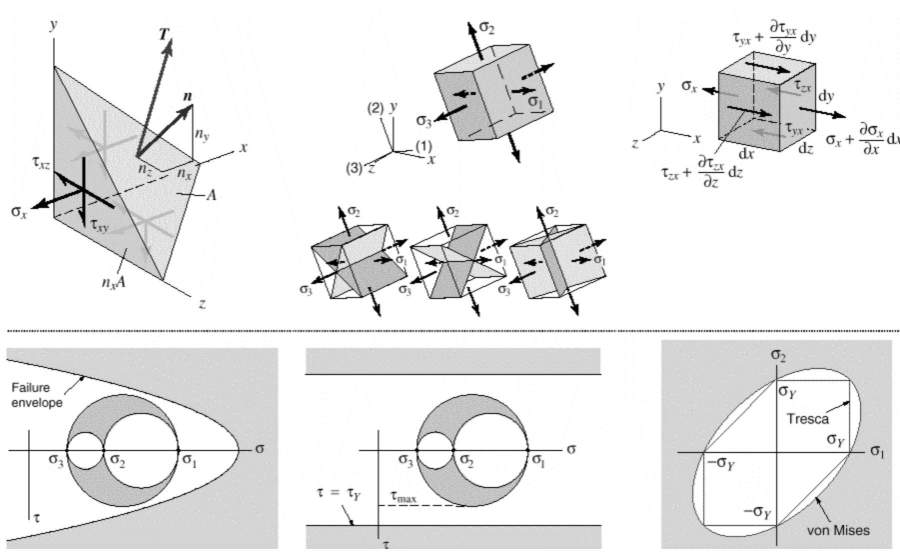
Stress and Strain



Courtesy of: http://moe.tam.uiuc.edu/Courses/TAM326/

Stress and Strain

$$s_a = \frac{F_N}{A_c} = stress; \qquad e_a = \frac{dL}{L} = strain$$

$$s_a = E_m e_a = Hooke's\ Law; \qquad E_m = Young's$$ modulus of elasticity

2-D State of Stress-Strain :

$$e_y = \frac{s_y}{E_m} - n_p \frac{s_x}{E_m}; \qquad e_x = \frac{s_x}{E_m} - n_p \frac{s_y}{E_m}$$

$$s_x = \frac{E_m(e_x + n_p e_y)}{1 - n_p}; \qquad s_y = \frac{E_m(e_y + n_p e_x)}{1 - n_p}$$

$$t_{xy} = G g_{xy}$$

$$n_p = \frac{transverse\,strain}{axial\ strain} = \frac{e_t}{e_a} = Poisson's$$ ratio

Resistance Strain Gages

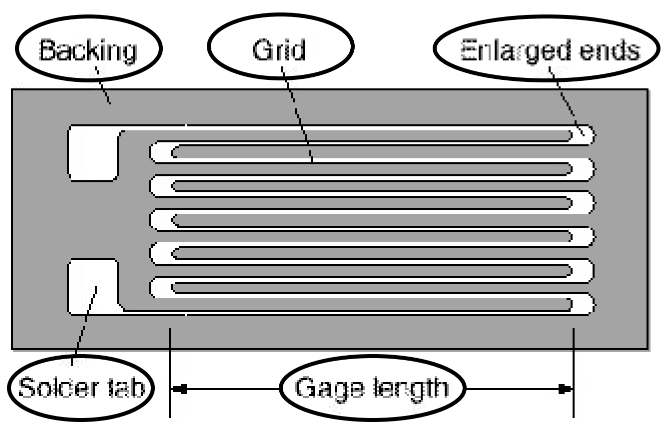


Fig. 1. Etched-foil bonded electrical resistance strain gage.

Gauge Factor

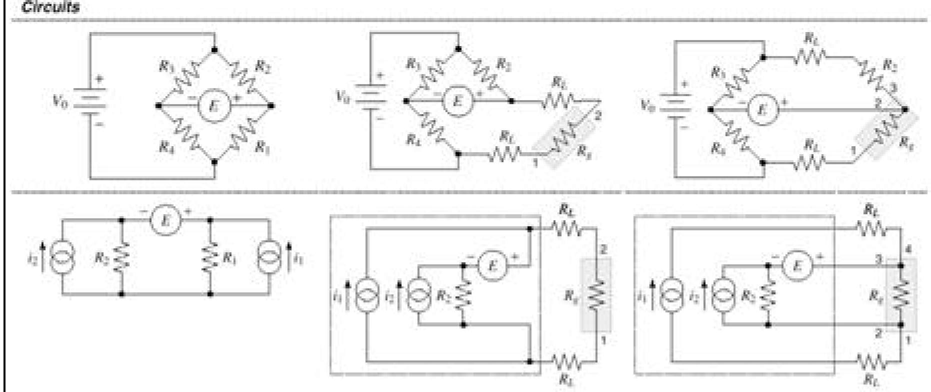
$$GF = \frac{output}{input} = \frac{dR/R}{e_a} = \frac{dR/R}{dL/L} = 1 + 2n_p + p_1 E_m$$

$$R = \frac{r_e L}{A_c} \Rightarrow \boxed{\frac{dR}{R} = \frac{dr_e}{r_e} + \frac{dL}{L} - \frac{dA_c}{A_c}} = \frac{dL}{L}\left(1 + 2n_p + p_1 E_m\right)$$

$$\text{where}: n_p = \frac{transverse\,strain}{axial\,strain} = \frac{e_t}{e_a} = \frac{-dD/D}{dL/L} = Poisson's \text{ ratio}$$

$$p_1 = \frac{1}{E_m}\frac{dr_e/r_e}{dL/L} = \text{piezoresistance coefficient}$$

Strain Gage Electrical Circuits



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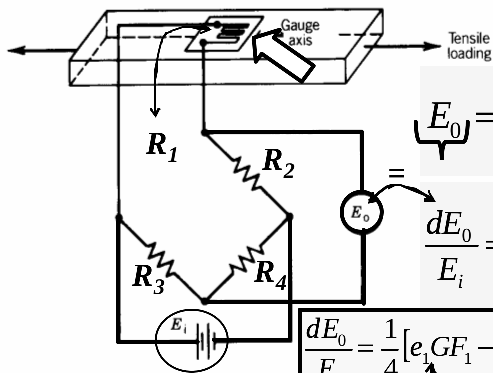
Electrical Circuits: Wheatstone Bridge

FIGURE 11.10
Strain gauge circuit subject to uniaxial tension.



For a "Quarter Bridge"

$$E_0 = E_i \frac{(R_1 + dR_1)R_4 - R_3 R_2}{(R_1 + dR_1 + R_2)(R_3 + R_4)}$$

$$= \quad \frac{dE_0}{E_i} = \frac{dR/R}{4 + 2\left(dR/R\right)} \approx \frac{dR/R}{4}$$

If all initial resistances were equal R

$$\frac{dE_0}{E_i} = \frac{1}{4}\left[e_1 GF_1 - e_2 GF_2 - e_3 GF_3 + e_4 GF_4\right] \text{ for full bridge}$$

$dR_n = 0$ for fixed n^{th}-resistor, then $e_n = GF_n = 0$

[*Eq.*11.25, *p.*504 *in Text*]

Note:

$$GF = \frac{dR/R}{e} \Rightarrow e\,GF = dR/R$$

Bridge Constant

$$k = \frac{dE_{0,any}}{dE_{0,sngl}} = \frac{\left(dE_{0,any}/E_i\right)}{\left(dE_{0,sngl}/E_i\right)} = \text{bridge constant}$$

$$\frac{dE_{0,any}}{E_i} = \frac{1}{4}\left[e_1 GF_1 - e_2 GF_2 - e_3 GF_3 + e_4 GF_4\right], \text{ for } any \text{ case}$$

$$\frac{dE_{0,sngl}}{E_i} = \frac{1}{4} e_1 GF, \text{ for a single guage}$$

$$\text{NOTE : Bridge sensitivit y is : } K_B = \frac{dE_0}{dR}\left[\frac{V}{\Omega}\right] \neq k$$

Problem 11.7

PROBLEM 11.7

KNOWN: A strain gauge with $R_1 = 120$, $GF = 2$ in an equal arm Wheatstone bridge $R_2 = R_3 = R_4 = 120\ \Omega$. Maximum gauge current is 0.05 A

FIND: Maximum input bridge voltage

SOLUTION: From a basic circuit analysis, assuming infinite meter resistance

$$i_1 = \frac{E_i}{R_1 + R_2}$$

and $E_i = i_1(R_1 + R_2)$ $\quad$ $E_i = (0.05\ A)(240\ \Omega) = 12\ V$

Problem 11.10

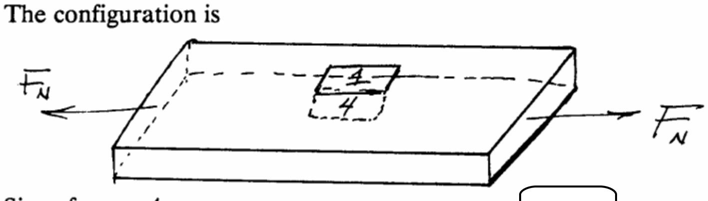
PROBLEM 11.10

KNOWN: A steel member (ν_p = 0.3) subject to simple axial tension. Strain gauges are mounted on top center, and bottom center.

FIND: Bridge constant, for gauge locations 1 and 4.

SOLUTION:

The configuration is



Since for any 4 gauges

$$\frac{\delta E_o}{E_i} = \frac{GF}{4}(\varepsilon_1 - \varepsilon_2 + \varepsilon_4 - \varepsilon_3)$$

$\varepsilon_4 = \varepsilon_1$

=0, fixed resistors

and for a single gauge, sensing maximum strain

$$\frac{\delta E_o}{E_i} = \frac{GF}{4}(\varepsilon_{max})$$

In the present case both gauges sense the maximum strain, and the outputs are additive $K = 2$ ∘∘∘ κ=?

δE_O = 10 mV and E_i = 10 V R = 120 Ω and , GF = 2

For the gauge sensing maximum strain

$$\frac{\delta E_o^s}{E_i} = \left[\frac{\varepsilon_{max} GF}{4 + 2\varepsilon_{max} GF}\right]$$

The actual output is then

$$\delta E_o = K \delta E_o^s$$

Solving for ε_{max} yields

$$\varepsilon_{max} = 0.001$$

Therefore

$$\varepsilon_{axial} = \varepsilon_{max} = 0.001$$

$$\varepsilon_t = 0.0003$$

$e_t = n_p e_{axial}$

Problem ~~11.19~~ (new 11.26)

PROBLEM ~~11.19~~ (new 11.26

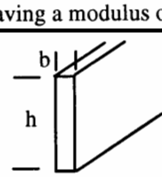
KNOWN: It is desired to design a strain gauge based scale, using a cantilever beam. The beam is 21 cm long, 0.4 cm thick, and 2 cm wide. The loads are up to 200 g, applied 20 cm from the fixed end of the beam. The required uncertainty level is 4%.

FIND: Design measurement system, including strain gauge placement, bridge characteristics, and signal conditioning.

SOLUTION: Because this design problem has a wide variety of solutions, a general approach and some representative equations and results will be provided.

The beam is made of 2024–T4 Aluminum, having a modulus of 71 GPa. For a cantilever beam, the deflection at the free end is

$$f = \frac{Wl^3}{3EI} \qquad I = \frac{bh^3}{12}$$



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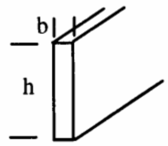

where f is the deflection, W is the load, l is the distance from the fixed end to the load, I is the moment of inertia, and E is the modulus. A representative design may be examined by assuming a bridge having a single active gauge sensing the maximum axial strain. This would correspond to a location on the surface of the beam at the location where the load is applied. In this case the deflection f is

$$f = \frac{Wl^3}{3EI} = \frac{(1960)(0.2)^2}{3(71 \times 10^9)(2.67 \times 10^{-9})} = 2.7cm \qquad I = \frac{bh^3}{12} = \frac{(0.004)(0.02)^3}{12} = 2.67 \times 10^{-9}$$

$$\sigma_x = \frac{Wl(h/2)}{I} = \frac{(1960)(0.2)(0.01)}{(2.67 \times 10^{-9})} = 1.47 \times 10^9 Pa$$

Then with E = 71 GPa $\varepsilon = \frac{\sigma_x}{E} = \frac{1.47 \times 10^9}{71 \times 10^9} = 0.0207$ $\quad \frac{\delta R}{R} = \varepsilon GF = 0.0207 \times 2 = 0.0414$

$$\frac{\delta E_o}{E_i} = \frac{\delta R/R}{4} = \frac{0.0414}{4} = 0.01$$

12 bit

$(\delta R/R)/\varepsilon = GF$

From these relationships, the output for a bridge excitation of 5 V is about 50 mV, which would create an uncertainty from the A/D resolution of about 2.4%. The bridge can be designed in a reasonable manner to meet the uncertainty constraint.