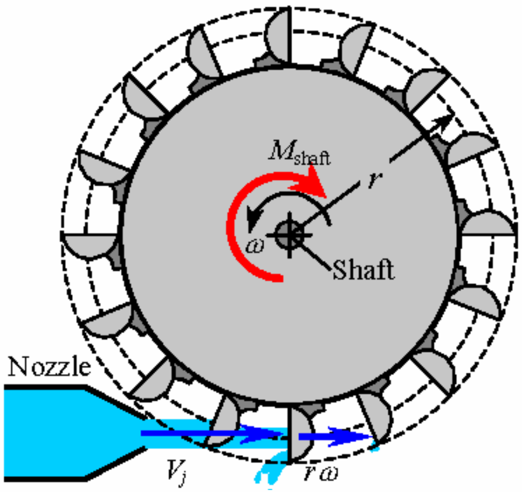
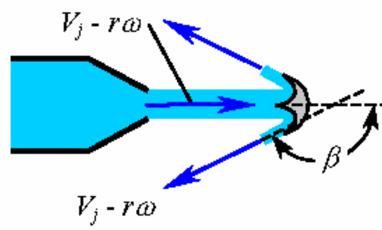
PROBLEM 6-51 Solution: <u>Pelton Wheel</u>
(Fluid Mechanics by Cengel & Cimbala, McGraw Hill, © 2006)

Kostic's Comments

(www.kostic.niu.edu; if used publicly please acknowledge this discussion)

The given Textbook Problem 6-51 solution reads:





Analysis The tangential velocity of buckets corresponding to an angular velocity of $\omega = 2\pi\dot{n}$ is $V_{bucket} = r\omega$. Then the relative velocity of the jet (relative to the bucket) becomes

$$V_r = V_j - V_{bucket} = V_j - r\omega$$

We take the imaginary disk that contains the Pelton wheel as the control volume. The inlet velocity of the fluid into this control volume is V_r, and the component of outlet velocity normal to the moment arm is $V_r\cos\beta$. The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{shaft} = r\dot{m}V_r\cos\beta - r\dot{m}V_r \quad \text{or} \quad M_{shaft} = r\dot{m}V_r(1-\cos\beta) = r\dot{m}(V_j - r\omega)(1-\cos\beta)$$

Kostic's COMMENTS:

In the above Pelton Wheel solution the Control Volume (CV) is taken to be a <u>rotating disk together with the Pelton wheel</u>, and the fluid velocities used were the <u>relative velocities</u> with regard to the rotating disk. This is <u>different approach</u> from the given Example 6–9: Power Generation from a Sprinkler System on p.257, where the CV is chosen to be a <u>stationary disk</u> with rotating nozzles inside it, and therefore the <u>absolute fluid velocities</u> were used. The students may be confused due to different approaches for the two similar problems: why the stationary CV disc and absolute velocities for the sprinkler, and the rotating disk and relative velocities for the Pelton wheel? Furthermore, if the approaches are interchanged the solution for the sprinkler will be wrong and for the Pelton wheel the same, i.e. correct, see below:

The Pelton Wheel, Pr.6-51	Inlet	Outlet
Relative tangential velocities, V_{t_rel} :	$V_j - r \cdot w$	$(V_j - r \cdot w)\cos b$
Absolute tangential velocities, $V_t = V_{t\_abs}$:	V_j	$(V_j - r \cdot w)\cos b + r \cdot w$

If we assume absolute stationary CV disk and use absolute water velocities (as is done for the sprinkler in Example 6-9), we will have:

$$\underbrace{M_t}_{\substack{\text{around}\\ \text{axis}}} = \sum_{OUT} \dot{m} r V_t - \sum_{IN} \dot{m} r V_t$$

$$\underbrace{\begin{aligned}(-M_{shaft}) &= \{\dot{m} r[(V_j - r \cdot w)\cos b + r \cdot w]\}_{OUT} - \{\dot{m} r[V_j]\}_{IN} \\ &= -\dot{m} r (V_j - r \cdot w)(1 - \cos b)\end{aligned}}_{\text{The same result as if relative velocities are used}}$$

When a moving (rotating) CV and a coordinating system attached to it is used with relative OUT and IN velocities, then the acceleration of the moving CV with regard to the stationary inertial system must be used, namely acceleration due to motion of the moving system origin (the translational acceleration, if any) and due to rotation $\vec{\Omega}$ (the angular, Coriolis and centripetal acceleration, if any). For Problem 6-51 there is only centripetal and Coriolis acceleration but both in radial direction, thus their tangential components are zero (note that angular acceleration is zero for constant rotational speed and the Coriolis's tangential component is zero if fluid within the CV does not have radial velocity component anywhere), thus relative velocities could be used in such case, like in Pr. 6-51. However, in the sprinkler Example 6-9 the fluid moves radially within the CV and the Coriolis acceleration $(2\vec{\Omega} \times \vec{V})$ contributes to the tangential component of the angular momentum equation, so using relative velocities without accounting for the Coriolis acceleration will produce the wrong result.

NOTE: I have not checked all solutions with this regard.