Flow with No External Forces (or Moments)

Fluid Mechanics, p.238 (or 253) by Cengel & Cimbala, McGraw Hill, © 2006
Summary (**Fluid Mechanics,** p. 238-239 and Summary, p.259)

Kostic's Comments

(www.kostic.niu.edu; if used publicly please acknowledge this discussion)

The Fluid Mechanics Textbook and Eq. (6-29) on page 239 read:

> When the mass m of the control volume remains nearly constant, the first term of the Eq. 6–28 simply becomes mass times acceleration since
>
> $$\frac{d(m\vec{V})_{CV}}{dt} = m_{CV}\frac{d\vec{V}_{CV}}{dt} = (m\vec{a})_{CV}$$
>
> Therefore, the control volume in this case can be treated as a solid body, with a net force or **thrust** of
>
> *Thrust:* $$\vec{F}_{body} = m_{body}\vec{a} = \sum_{in}\beta\dot{m}\vec{V} - \sum_{out}\beta\dot{m}\vec{V} \qquad \textbf{(6–29)}$$
>
> acting on the body. This approach can be used to determine the linear acceleration of space vehicles when a rocket is fired (Fig. 6–19).

There is ambiguity in nomenclature since $\vec{F}_{body}$ is used for (external) force acting on control volume (CV; zero in this case) in Eq.(6-4) on p.230 and subsequently in many other equations (e.g. Eq 6-11). I propose to use subscript "trust," that is $\vec{F}_{thrust}$, since it is a thrust force from the departing (or imparting) part of the "whole fixed-mass system" on which no external forces are acting (like a rocket jet engine or firing a bullet). When a part of a fixed mass system is fractured (departing from each other) then the momentum flaxes exert the action/reaction forces on each other. The physical explanation is given by relevant labeling of the momentum equation (Eq. 6-28, p. 238) below:

$$\underbrace{\left(\sum \vec{F} = 0\right)}_{\substack{\text{sum of}\\ \text{external forces}\\ \text{on whole system}}} = \underbrace{\underbrace{\frac{d(m\vec{V})_{CV}}{dt}}_{\vec{F}_{thrust}} + \underbrace{\left(\sum_{OUT} b\dot{m}\vec{V} - \sum_{IN} b\dot{m}\vec{V}\right)_{CS}}_{\text{net-departing mass momentum flux}}}_{\text{change of momentum rate for the total fixed-mass system (=0)}}$$

Therefore, from the above equation:

$$\underbrace{\vec{F}_{thrust}}_{\substack{\text{resultant}\\ \text{departing thrust force}\\ \text{on remaining part in CV}}} = \underbrace{\frac{d(m\vec{V})_{CV}}{dt}}_{\vec{F}_{thrust}} \approx (m\cdot\vec{a})_{CV} = \underbrace{-\left(\sum_{OUT} b\dot{m}\vec{V} - \sum_{IN} b\dot{m}\vec{V}\right)_{CS}}_{\text{negative net-departing mass momentum flux}}$$

Or in short:

$$\vec{F}_{thrust} \approx (m\cdot\vec{a})_{CV} = \sum_{IN} b\dot{m}\vec{V} - \sum_{OUT} b\dot{m}\vec{V}$$

NOTE: The remaining mass in the CV exerts pushing force onto the departing mass equal to and in opposite direction (sense) to the thrusting force of the departing mass onto the mass in the CV (action and reaction):

$$\underbrace{\vec{F}_{thrust}}_{\substack{\text{Departing mass}\\ \text{force onto the CV}}} = -\underbrace{\vec{F}_{push}}_{\substack{\text{CV force onto}\\ \text{the departing mass}}}$$