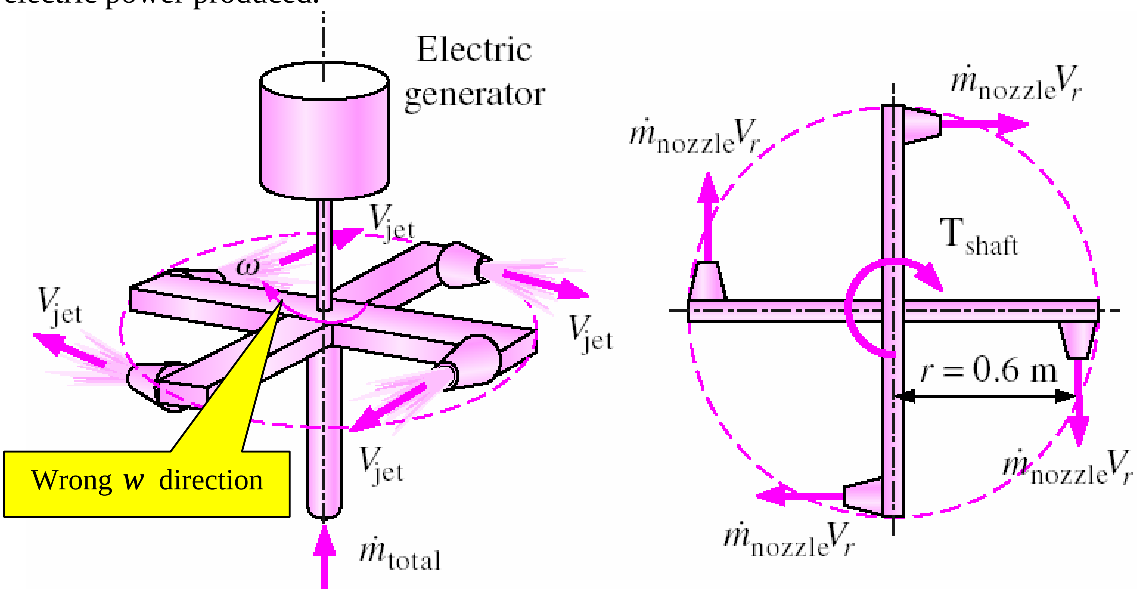
EXAMPLE 6–9: **<u>Power Generation from a Sprinkler System</u>**
(Fluid Mechanics, p.257 by Cengel & Cimbala, McGraw Hill, © 2006)

Kostic's Comments
(www.kostic.niu.edu; if used publicly please acknowledge this discussion)

"A large lawn sprinkler with four identical arms is to be converted into a turbine to generate electric power by attaching a generator to its rotating head, as shown in Fig. 6–38 (below). Water enters the sprinkler from the base along the axis of rotation at a rate of 20 L/s and leaves the nozzles in the tangential direction. The sprinkler rotates at a rate of 300 rpm in a horizontal plane. The diameter of each jet is 1 cm, and the normal distance between the axis of rotation and the center of each nozzle is 0.6 m. Estimate the electric power produced."



"… That is, the water in the nozzle is also moving at a velocity of 18.85 m/s in the opposite direction when it is discharged. Then the average velocity of the water jet relative to the control volume (or relative to a fixed location on earth) becomes:

$$V_r = V_{jet} - V_{nozzle} = 63.66 - 18.85 = 44.81 \text{ m/s}$$

… etc."

Kostic's COMMENTS:

On Fig. 6-38 (p.257 on left) the w direction should be in counterclockwise direction (opposite from $\boldsymbol{T}_{shaft}$, Fig. 6-38 on right). The velocity $\vec{V}$ in the angular momentum equation

Steady flow:
$$\sum \vec{M} = \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V} \qquad \textbf{(6–51)}$$

is the "absolute" velocity with reference to the inertial coordinate system (stationary CV with regard to earth) Thus, it is confusing to call it "relative to the CV" and use of

subscript "*r*" for relative, particularly since V_{jet} is water velocity relative to the moving nozzle. Furthermore, subscript "*r*" may imply the radial or relative velocity component which V_r above is not (thus confusing for students).
Actually, it is well known to students (from kinematics) that:

$$\underset{water}{\vec{V}_{abs}} = \vec{V}_{nozzle} + \underbrace{\vec{V}_{jet}}_{\substack{\text{relative}\\ \text{to nozzle}}}$$

For the rotating systems (like this sprinkler example) we use cylindrical coordinate system and the tangential component of the above vector equation becomes:

$$\underbrace{V_{abs,t}}_{-V_r \text{ on p.258}} = \underbrace{V_{nozzle,t}}_{\text{carrier}} + \underbrace{V_{jet,t}}_{\substack{\text{relative}\\ \text{over carrier}}} = +\left|V_{nozzle}\right| + \left(-\left|V_{jet}\right|\right) = \underbrace{V_{nozzle} - V_{jet}}_{\substack{\text{using magnitudes}\\ \text{(oposit signs from Text, p.258)}}}$$

The radial components of the above vector equation are irrelevant (result in zero momentum around axis of rotation). Then, the *z*-axis component (i.e. the tangential/circumferential component around *z*-axis in counterclockwise direction as pozitive, the only rotation mechanically allowed by the system) of the vector Eq. (6-54) (this should be explicitly stated) becomes:

$$\underbrace{M_t}_{\substack{\text{around}\\ \text{z-axis}}} = \sum_{OUT} \dot{m} r V_t - \sum_{IN} \dot{m} r V_t \quad \text{or} \quad \underbrace{\left(-T_{shaft}\right) = \left(\dot{m} r V_{abs,t}\right)_{OUT} - (0)_{IN} = \dot{m} r\left(V_{nozzle} - V_{jet}\right)}_{\substack{\text{The same as the Textbook result (opposite sign twice),}\\ \text{but this emfasize the physics (water absolute tangential velcity)}}}$$

Additional (Kostic's) comments:

FM, p. 258: For the second limiting case (T_{shaft} =0, no electrical generator); it may be added to the *"The actual power produced will be less than this due to generator inefficiency (Chap. 5),"* that V_{nozzle} and V_{jet} are equal but in opposite direction (as if the nozzle is sliding backward over a stationary water) so that the water tangential absolute velocity (and thus torque) are zero and the water mass is "dropping down as a downward fountain" under gravity with zero angular momentum (around axis of rotation). This (T_{shaft} =0) is only possible for ideal frictionless nozzle-process (i.e. 100% nozzle efficiency, as a no-load ideal turbine), otherwise there will be a resisting torque due to friction of water, shaft and surrounding air.

FM, p. 259: *"The actual power produced will be less than this due to generator inefficiency (Chap. 5) [also in Essentials of FLUID MECHANICS, p. 191: "The actual power produced will be less than this due to generator inefficiency (Chap. 5) and other irreversible losses, such as shaft friction and aerodynamic drag."*
The fluid friction within the nozzle should be added, since it is usually larger than the mentioned shaft friction and aerodynamic drag, all of which contribute to the nozzle (as turbine) efficiency.