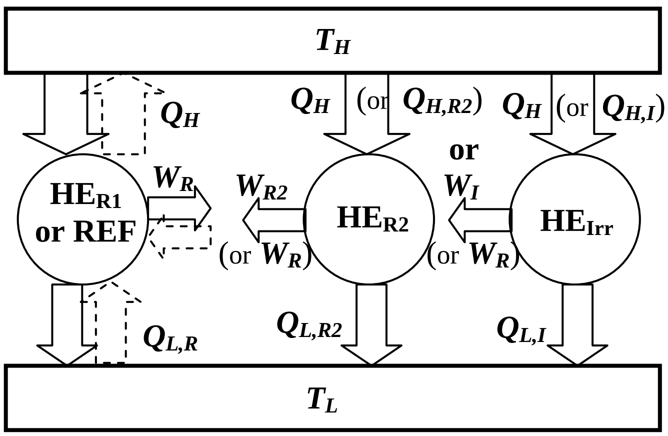
2nd Law Proof-Carnot Cycle *by M. Kostic, Northern Illinois University*

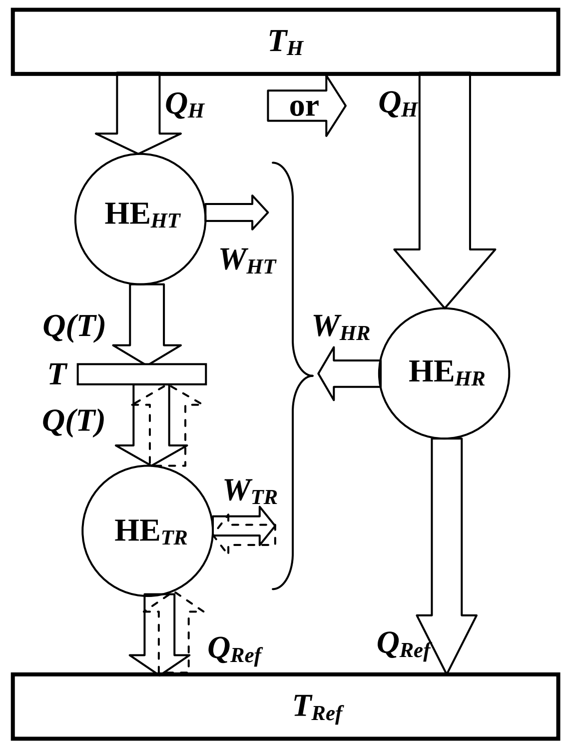


$$\text{If } \left(\eta_R = \frac{W_R}{Q_H}\right) < \left(\eta_{R2} = \frac{W_{R2}}{Q_H} = \frac{W_R}{Q_{H,R2}}\right) < \left(\eta_I = \frac{W_I}{Q_H}\right)$$

$$\text{and } Q_H = \text{constant, then: } W_R < W_{R2} < W_I$$

$$\text{and } Q_{L,R} > Q_{L,R2} > Q_{L,I} \quad (\text{or } Q_{H,R2/I} < Q_H)$$

Fig. X1: If a reversible heat engine (Rev. HE) has a smaller efficiency than other HE (Rev.2 or Irrev.), then if reversed (Refrigeration cycle) and combined with other HE, it will result in impossible net-work from a single reservoir ($W_{R2/I} - W_R$), or heat transfer from low to high temperature ($Q_H - Q_{H,R2/I}$).



$$Q(T) = Q_R f(T) \text{ for given } T_{\text{Ref}}$$

Fig. X2: For a fixed T_H, T_{Rref}, Q_H, and Q_{Ref}, the $Q(T)$ is proportional to Q_{Ref} (efficiency is intensive property) and an increasing function of T for a given T_{Ref}.

Z

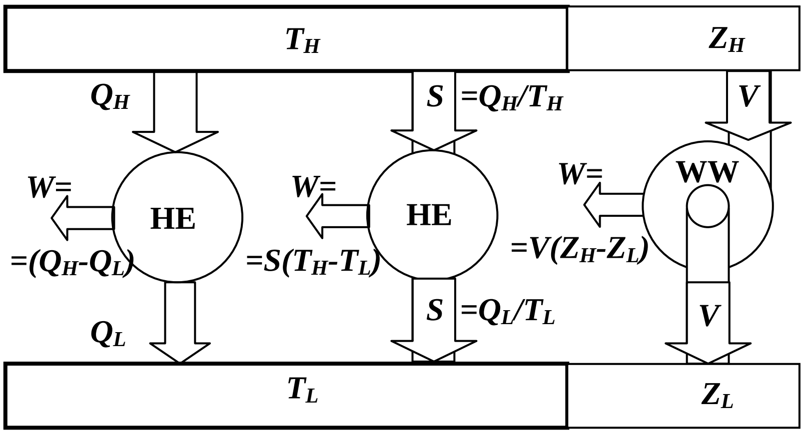


Fig. X3: Similarity between a heat engine (HE) and a water wheel (WW).

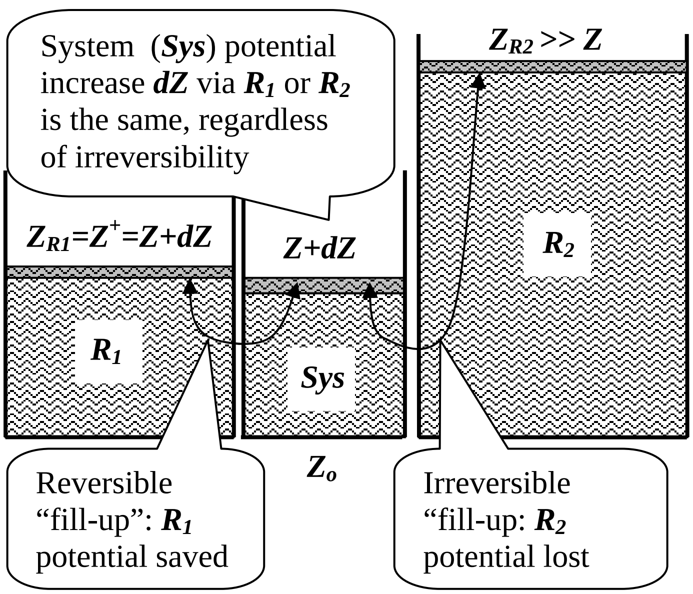
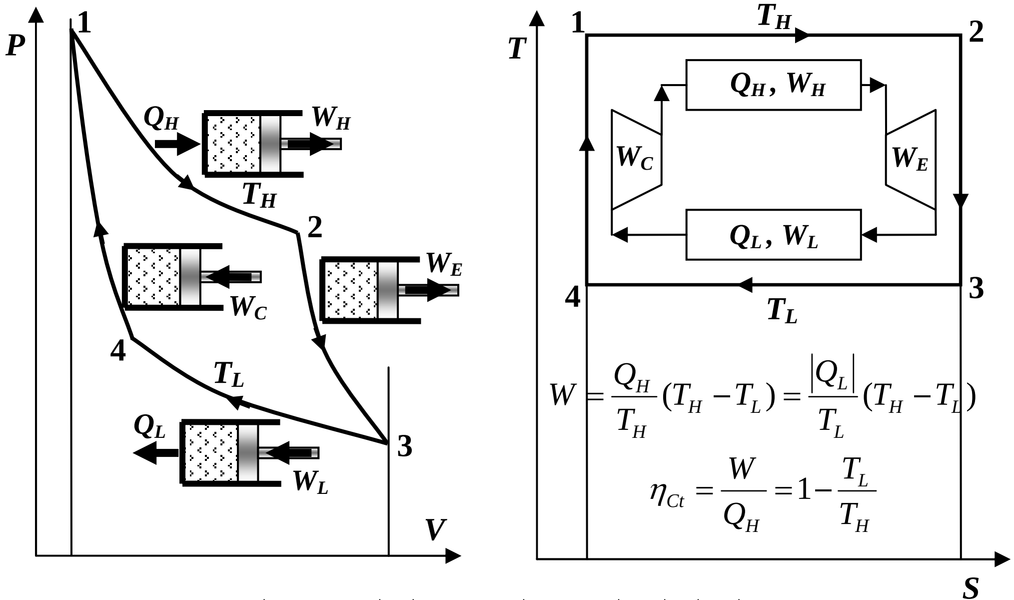


Fig. X4: Reversible process is at infinitesimal potential difference ($Z_{R1} – Z = dZ$) and irreversible process is at a finite potential difference ($Z_{R2} – Z >> 0$).



$$\eta_{Ct} = \frac{W}{Q_H} = \frac{|W_H + W_E| - |W_L + W_C|}{Q_H} \qquad = \frac{|Q_H| - |Q_L|}{Q_H} = 1 - \frac{T_L}{T_H}$$

Figure x5: **Heat engine ideal *Carnot* cycle: note thermal and mechanical expansions and compressions (the former is needed for net-work out, while the latter is needed to provide reversible heat transfer).**

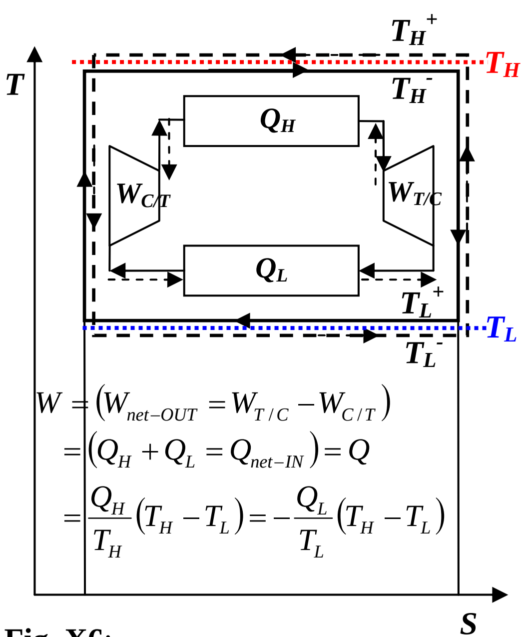


Fig. X6:
Reversible Heat-engine (solid lines) and Refrigeration (dashed lines, reversed directions) *Carnot* cycle.

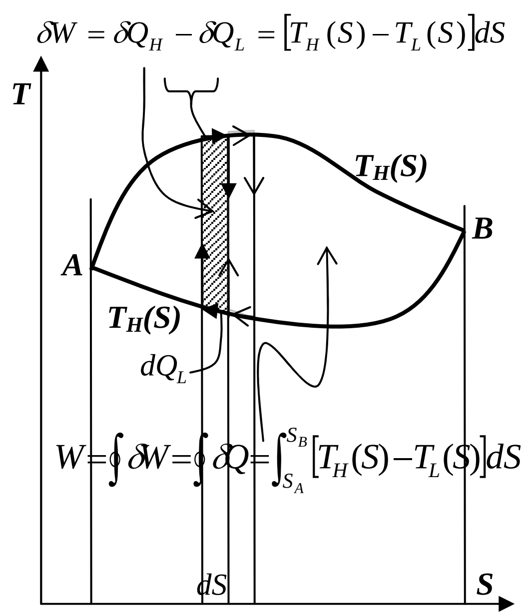
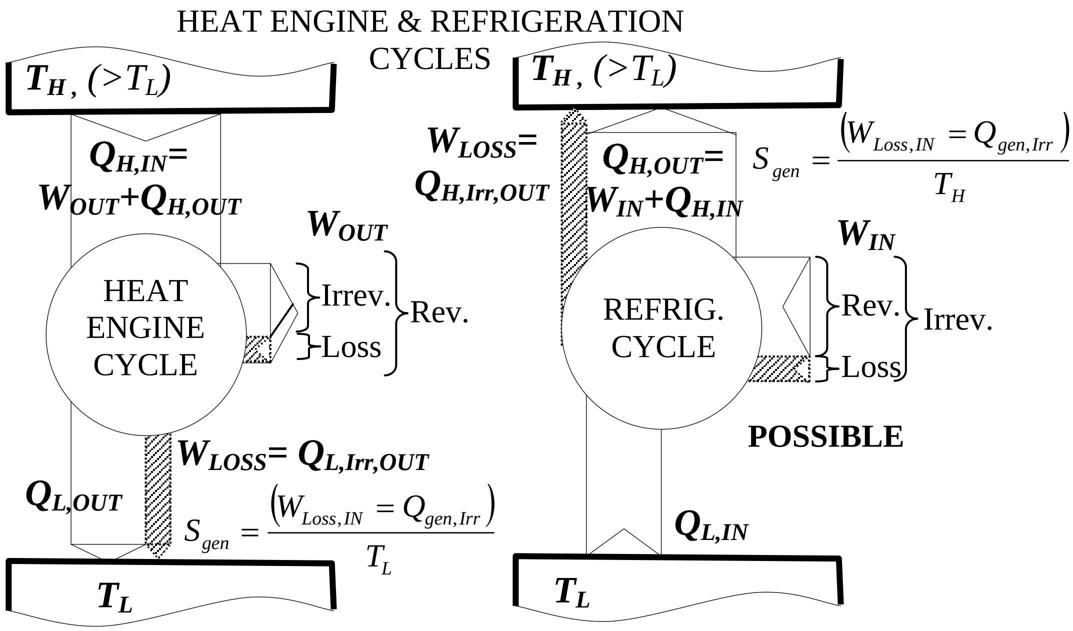
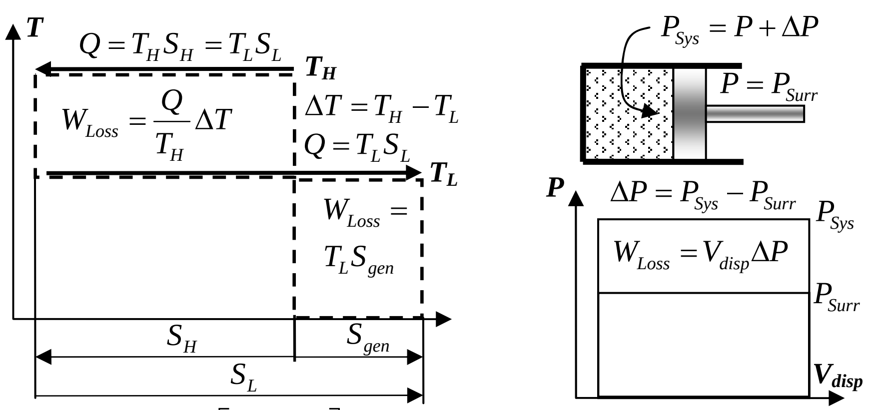


Fig. X7:
Variable temperature reservoirs require multi-stage Carnot cycles



$$\underbrace{W_{net-OUT}}_{-W_{net-IN}} = Q_{IN} - \underbrace{(\overbrace{Q_{rev,OUT} + \underbrace{Q_{gen,OUT}}_{W_{Loss,IN}}}^{Q_{OUT}})}_{Q_{net-IN}}$$

Fig. X8: Irreversible work-in-loss will results in more heat out, and thus in less work out or more work in than in the corresponding reversible cycle.



$$S_{gen} = \frac{W_{Loss}}{T_L} = \frac{1}{T_L}\left[\frac{Q}{T_H}\underbrace{\Delta T}_{T_H - T_L}\right] = S_L - S_H \qquad S_{gen} = \frac{W_{Loss}}{T} = \frac{1}{T}\left[V_{disp}\left(\underbrace{\Delta P}_{P_{Sys} - P}\right)\right]$$

Fig. X9: **Irreversibility and entropy generation due to process-potential differences (finite ΔT and ΔP, along with S_{gen}, must take place in finite space and time. Process is reversible if $\Delta T = \Delta P = 0$.**

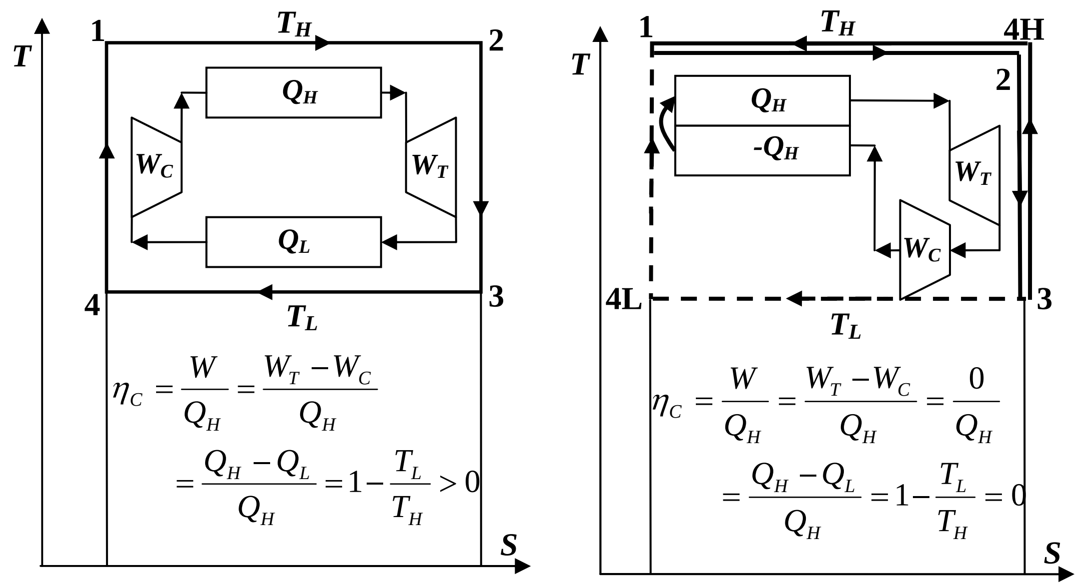


Figure X10: Heat engine ideal *Carnot* cycle between two different temperature heat-reservoirs ($T_H > T_L$ and $W > 0$) and with a single temperature heat-reservoirs ($T_H = T_L$ and $W = 0$, ideal reversible cycle). Low-temperature thermal compression is needed (critical), not the mechanical (isentropic) compression, to realize work potential between the two different temperature heat-reservoirs, due to internal thermal energy transfer via heat ($W = Q_H - Q_L > 0$). The isentropic expansion and compression are needed to provide temperature for reversible heat transfer, while net thermal expansion-compression provides for the net-work out of the cycle.

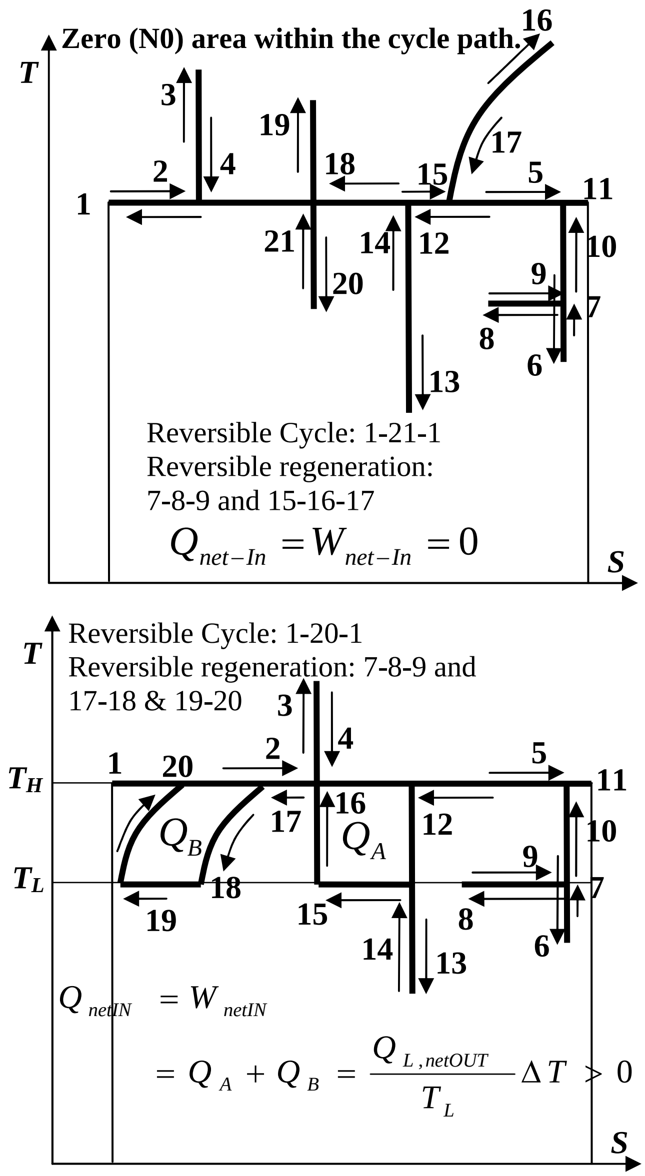


Figure X11: Arbitrary reversible thermo-mechanical cycle with a single constant-temperature reservoir (including reversible regeneration heat transfer): upon completion of the cycle the net heat and work transfer are zero (top).
Arbitrary reversible thermo-mechanical cycle between two constant-temperature reservoirs (T_H>T_L) and including reversible regeneration heat transfer: upon completion of the cycle the net heat and work transfer are equal and positive (bottom).
Low-temperature thermal compression is needed (critical), not the mechanical (isentropic) compression, to realize work potential between the two different temperature heat-reservoirs. The isentropic expansion and compression are needed to provide temperature for reversible heat transfer, while net thermal expansion-compression provides for the net-work out of the cycle.

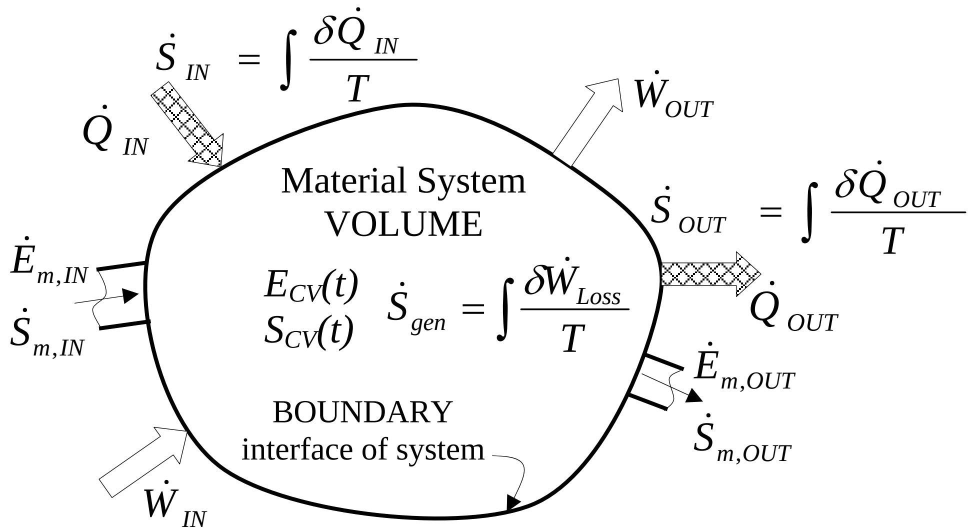


Fig. X12: Control-volume (CV) energy and entropy, and energy and entropy flows through the boundary interface of the control volume

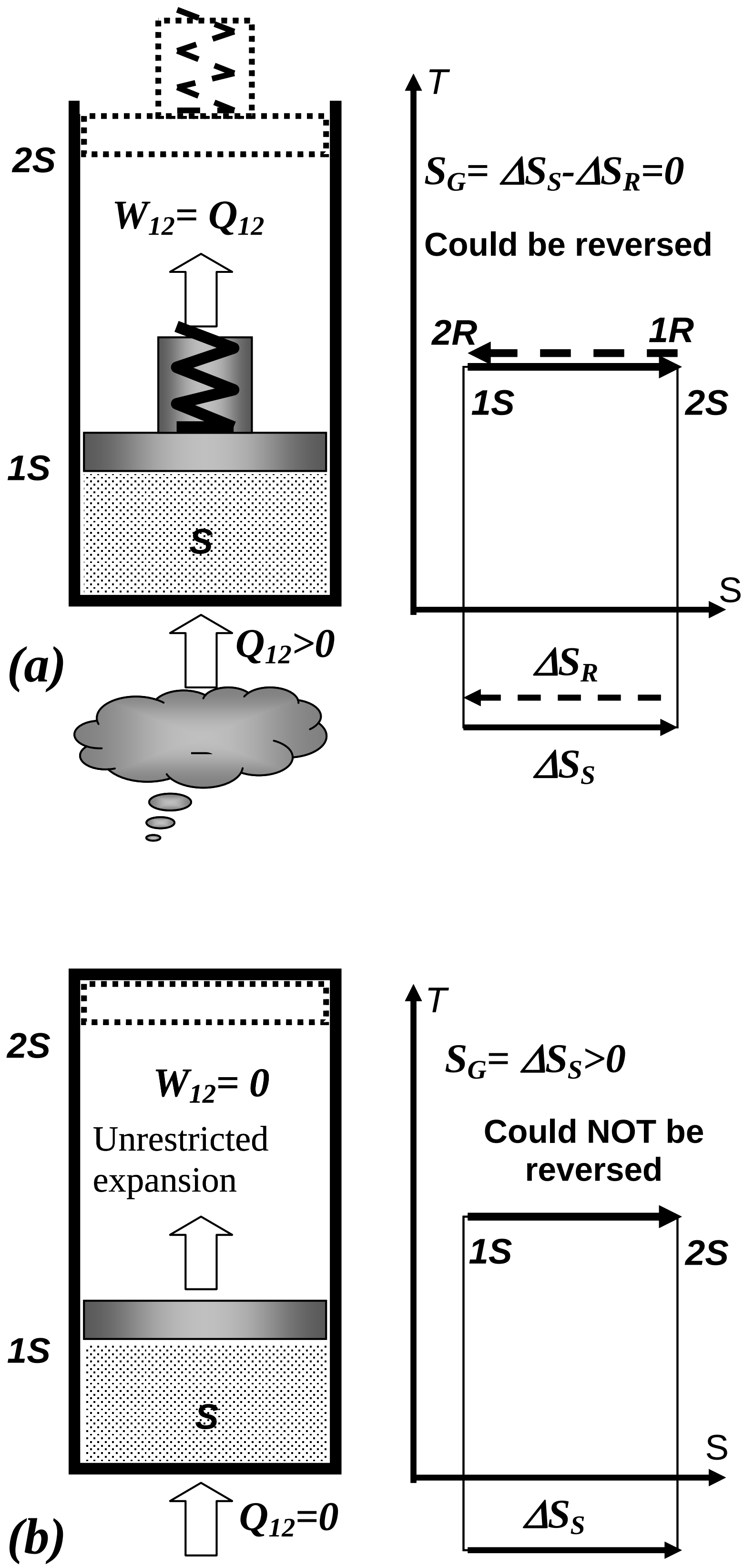
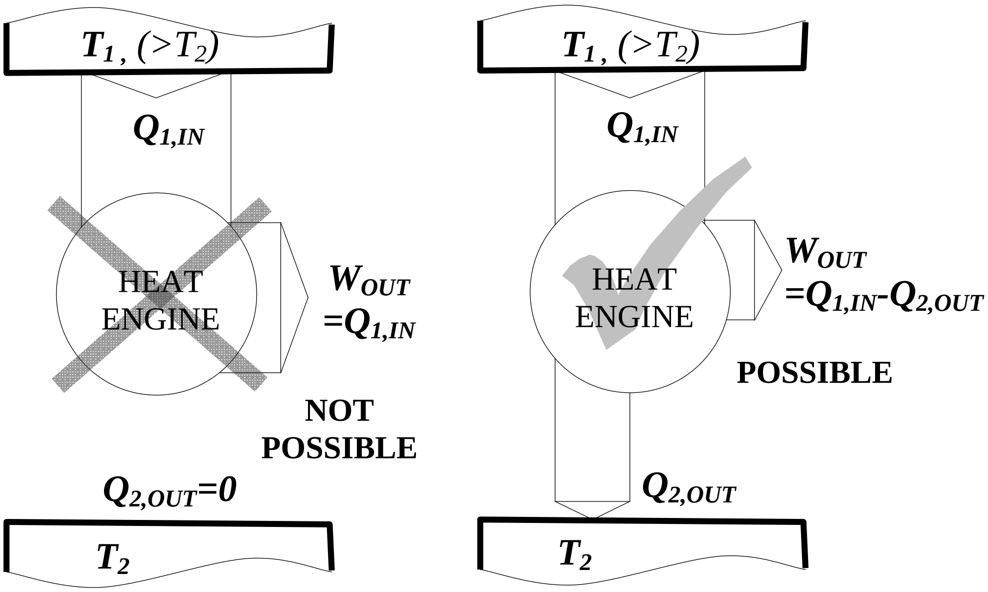
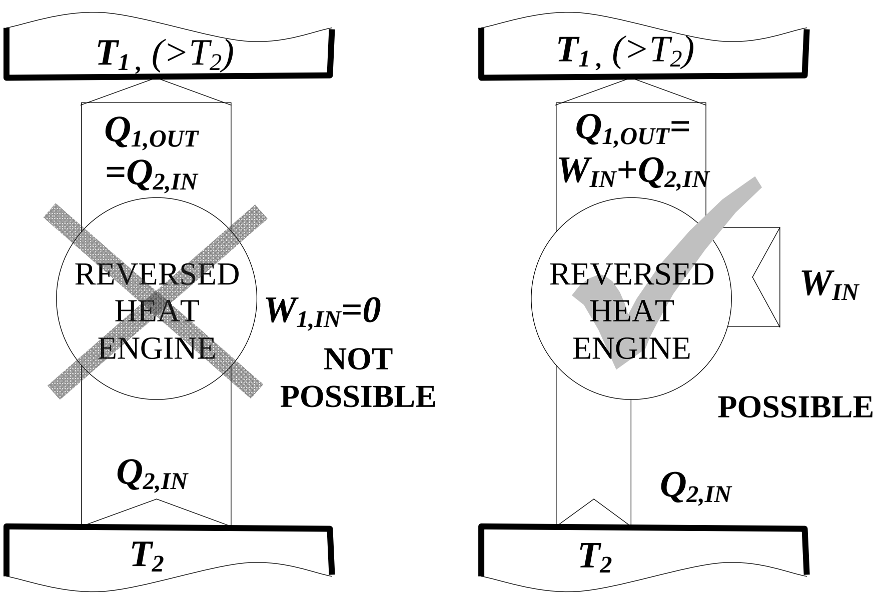


FIG. X13: (a) Isothermal reversible heat transfer and restricted reversible expansion; (b) adiabatic unrestricted irreversible expansion of the same initial system to the same final state.



(a) Kelvin-Plank statement of the Second Law



(b) Clausius statement of the Second Law

Fig. X14: The Second Law Statements:
(a) Kelvin-Plank, and (b) Clausius statements